



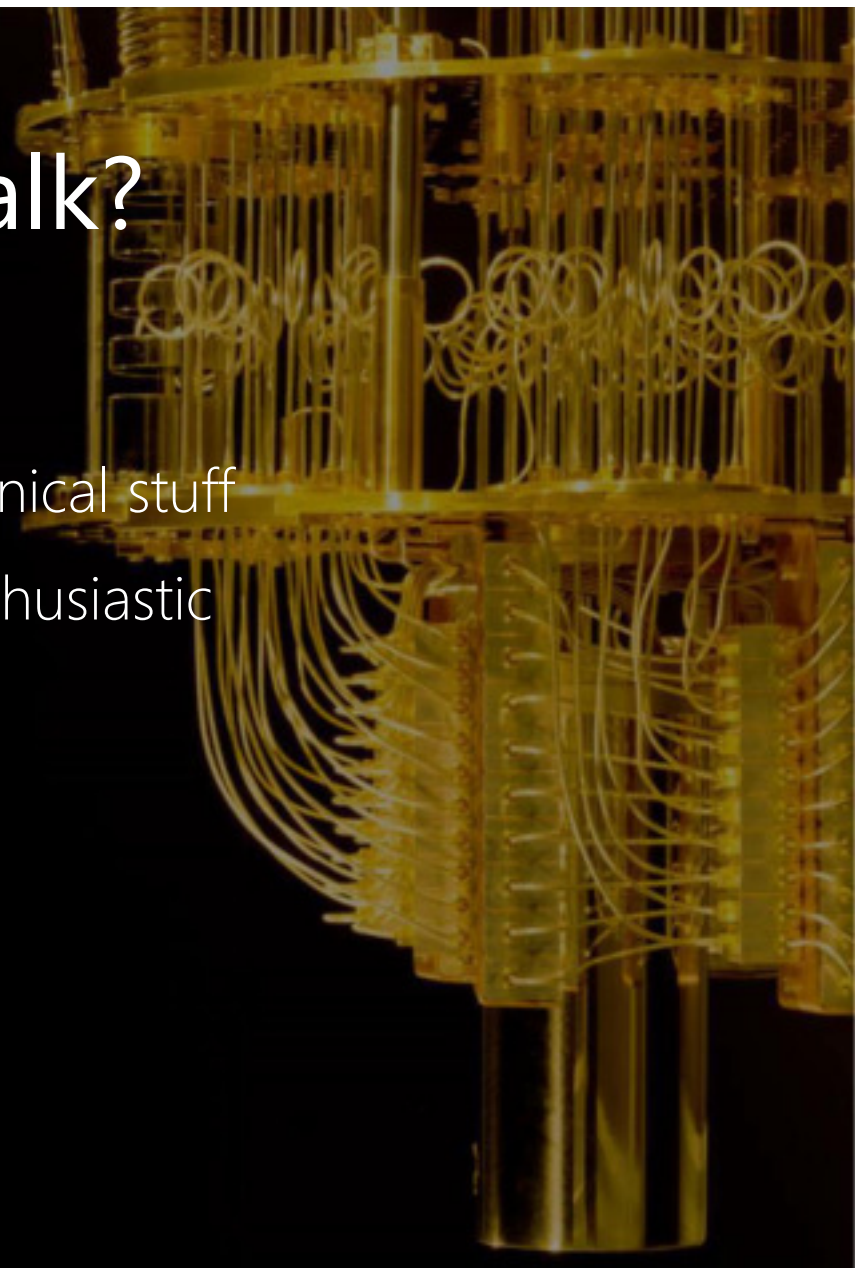
# **Quantum Computing**

## deep dive

Johnny Hooyberghs  
Robin Vercammen

# Why am I presenting this talk?

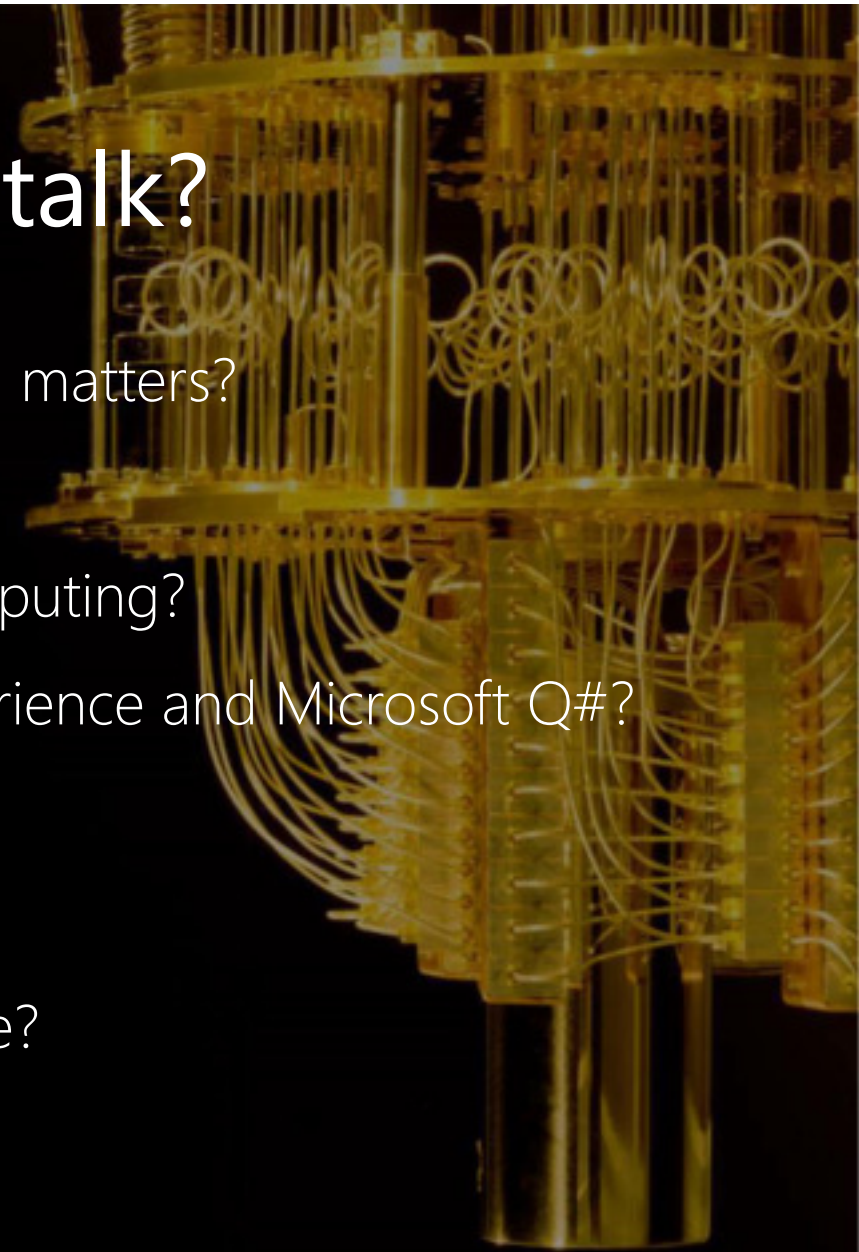
- As a human, I like to experiment and discover
- As a software engineer, I like to learn new technical stuff
- As a teacher, I like to teach and get people enthusiastic





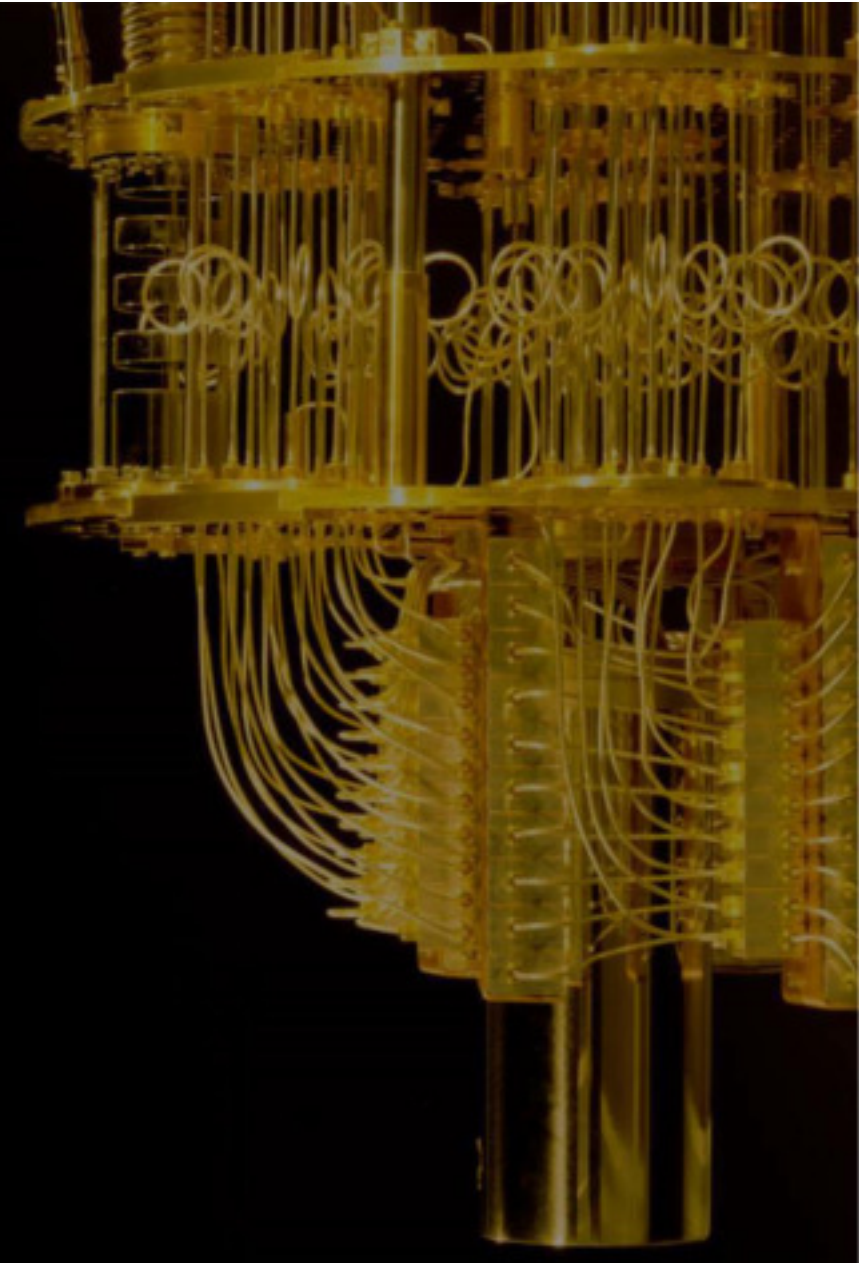
# What will you do after this talk?

- ☒ Be able to explain why quantum computing matters?
- ☒ Study more about quantum computing?
- ☒ Understand the basics about quantum computing?
- ☒ Run quantum algorithms using IBM Q Experience and Microsoft Q#?
- ☐ Decipher quantum algorithms?
- ☐ Use quantum computing tomorrow?
- ☐ Use quantum computing in the next decade?



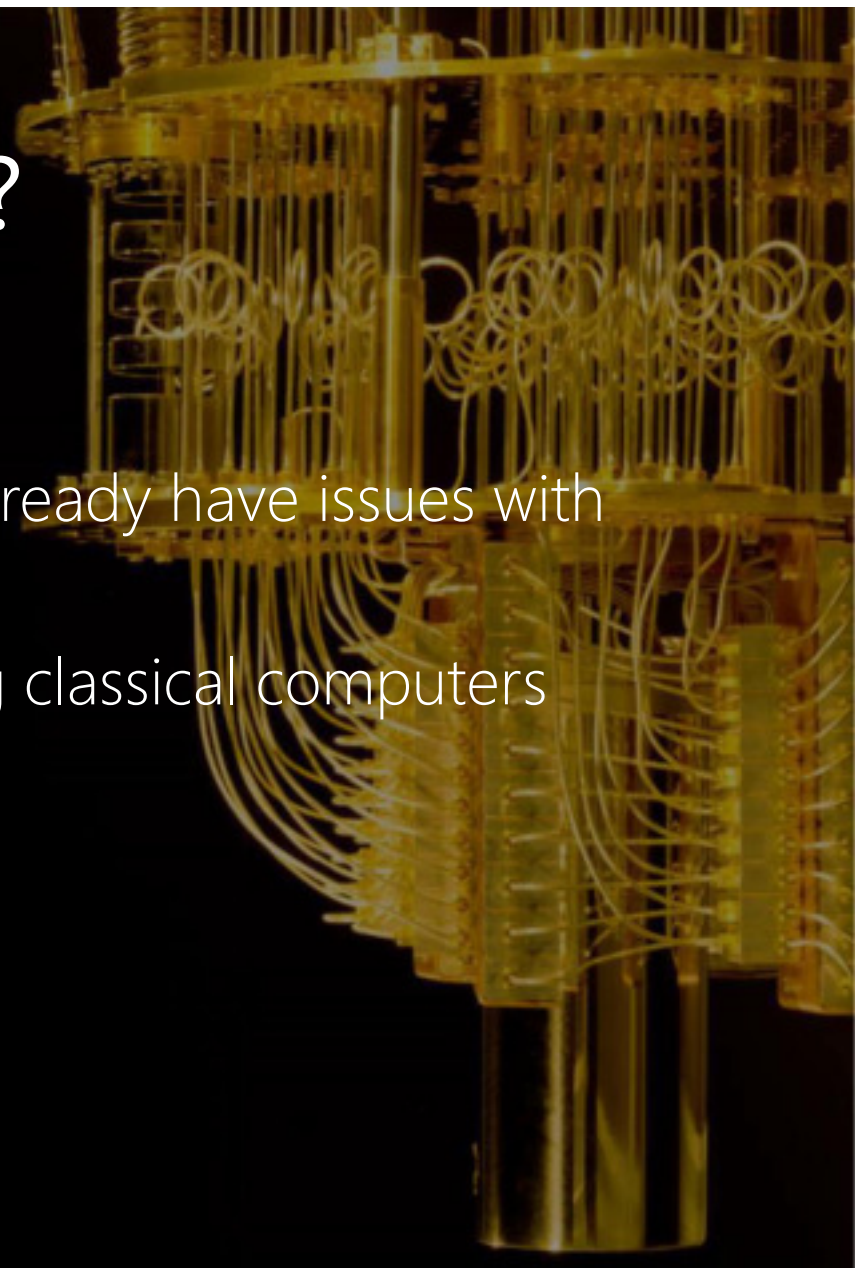
# Agenda

- Why Quantum Computing?
- Classic vs. Quantum
- Quantum superposition & entanglement
- Bit vs. Qubit
- IBM Q Experience
- Microsoft Q#
- Quantum Algorithms



# Why Quantum Computing?

- Moores law has its physical limits
- Current classical computing architectures already have issues with quantum effects because of their scale
- Why try to simulate a quantum world using classical computers

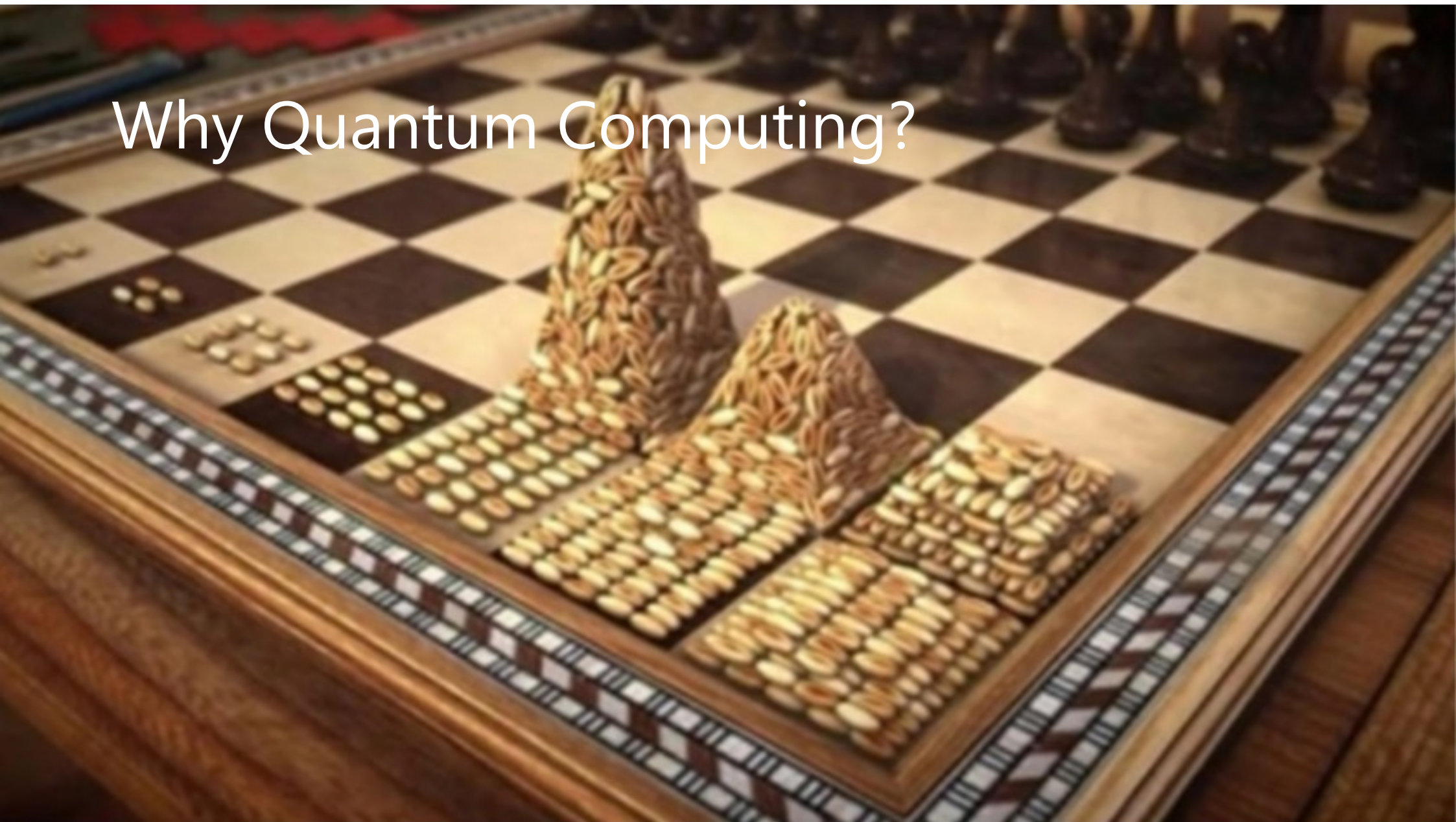


# Why Quantum Computing?





Why Quantum Computing?



# Classical vs. Quantum

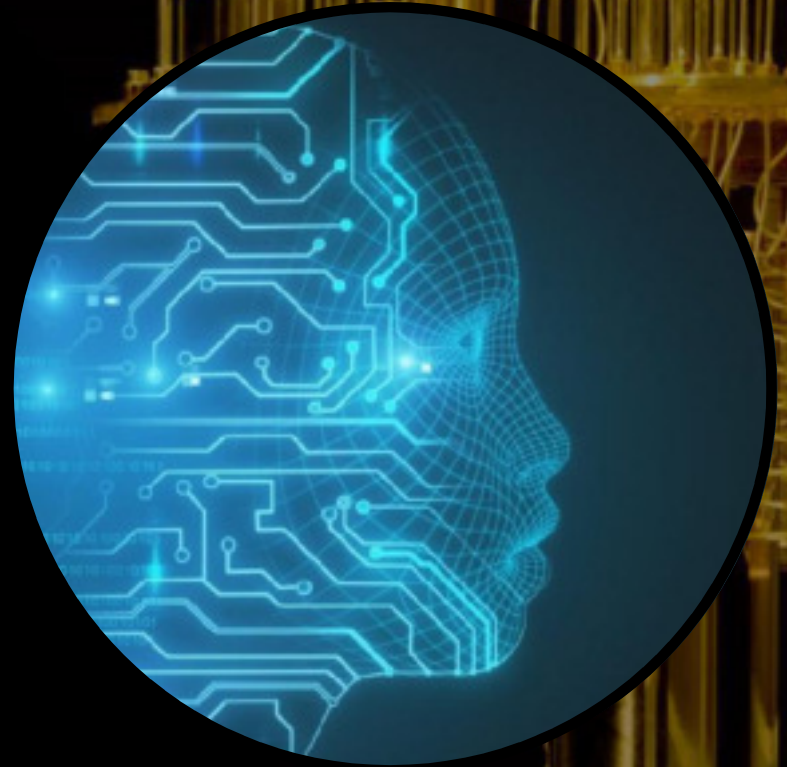
- Security
  - Public / private key encryption
  - Makes current encryption obsolete
  - QKD (Quantum Key Distribution)





# Classical vs. Quantum

- Artificial Intelligence
  - Analyze large quantities of data
  - Fast feedback
  - Emulate human mind



# Classical vs. Quantum

- Drug development
  - It takes a quantum system to emulate quantum mechanics
  - Interactions between molecules
  - Gene sequencing



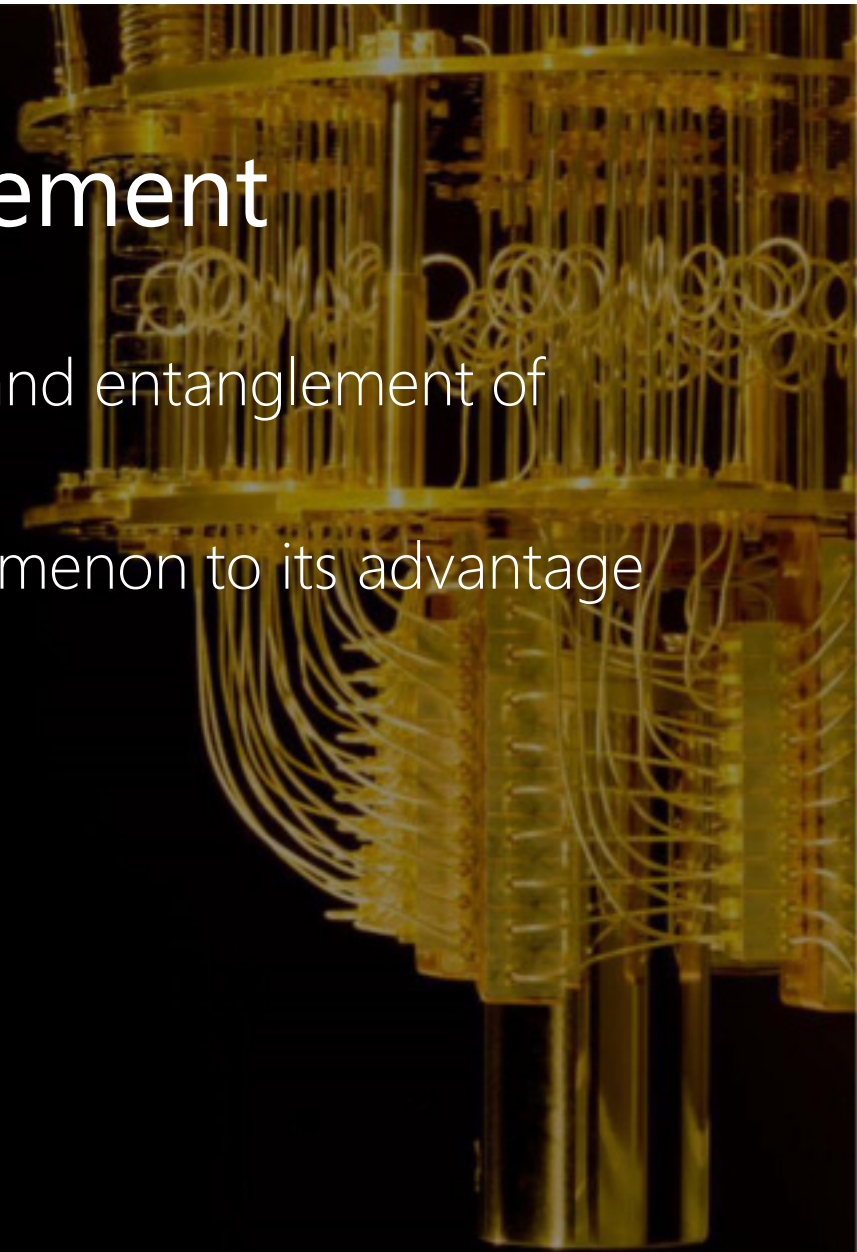


**CAN IT RUN CRYISIS?**

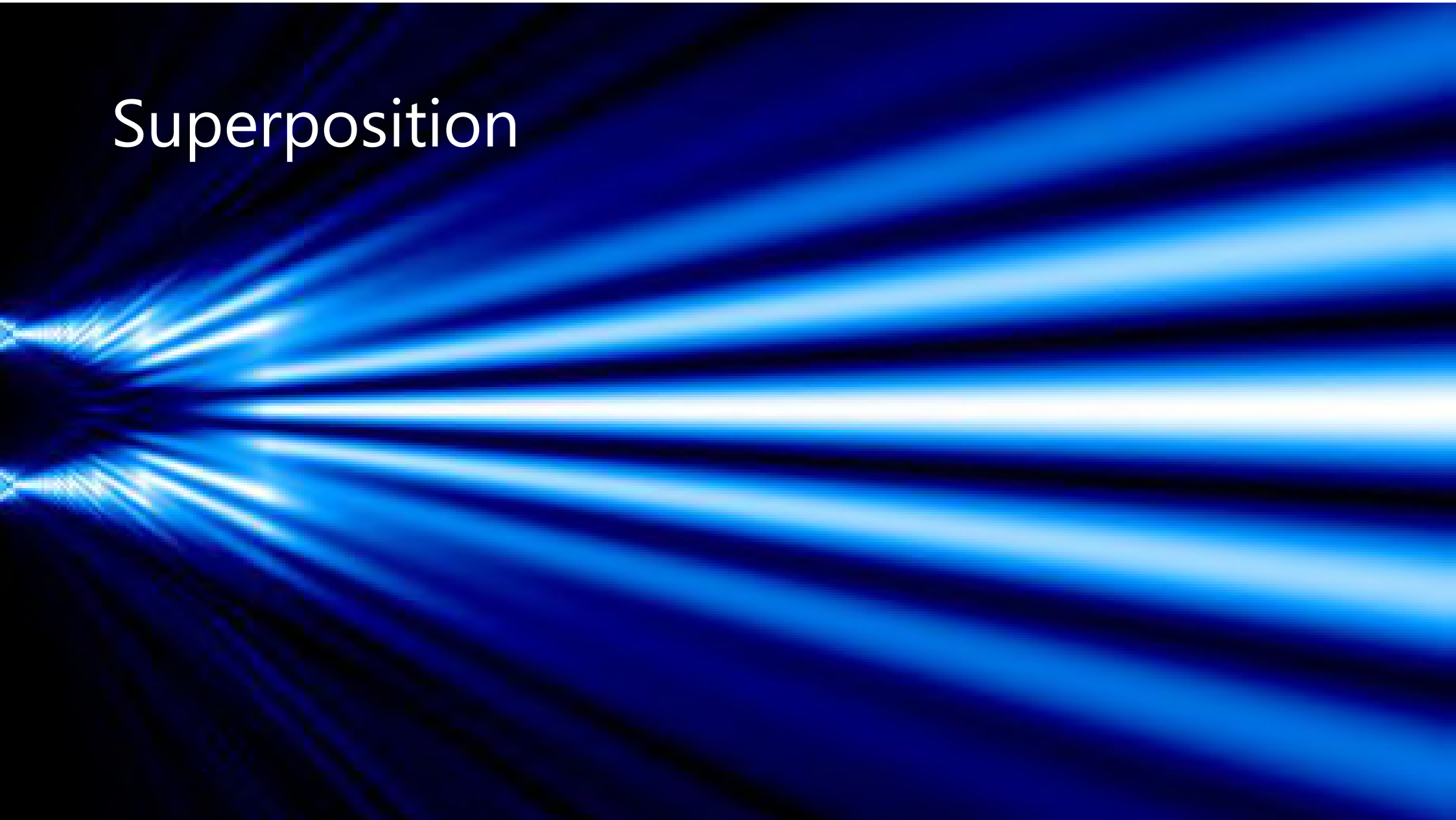


# Superposition and Entanglement

- Quantum Physics describes superposition and entanglement of quantum particles
- Quantum Computing can use these phenomenon to its advantage



Superposition

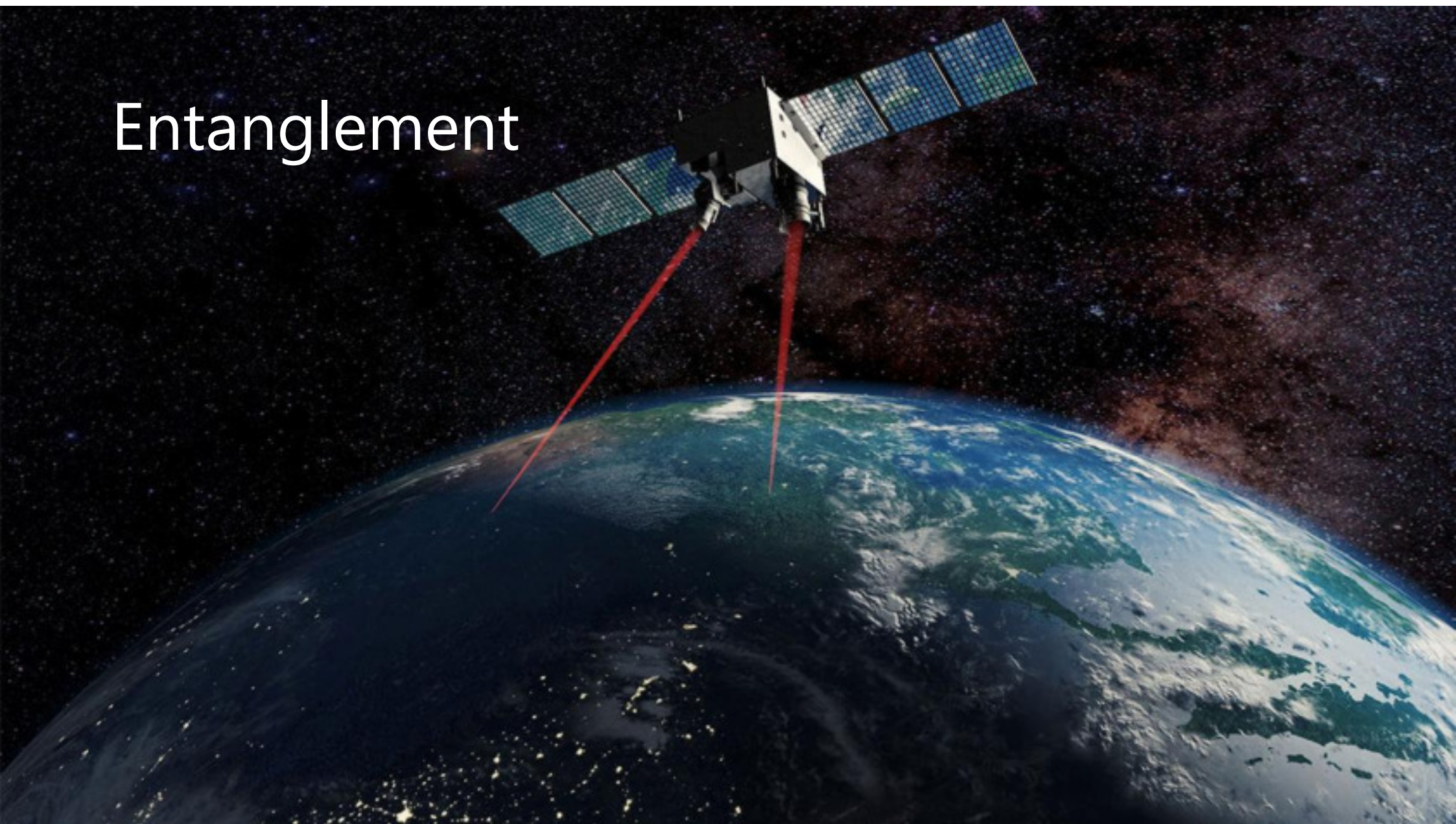


# Superposition





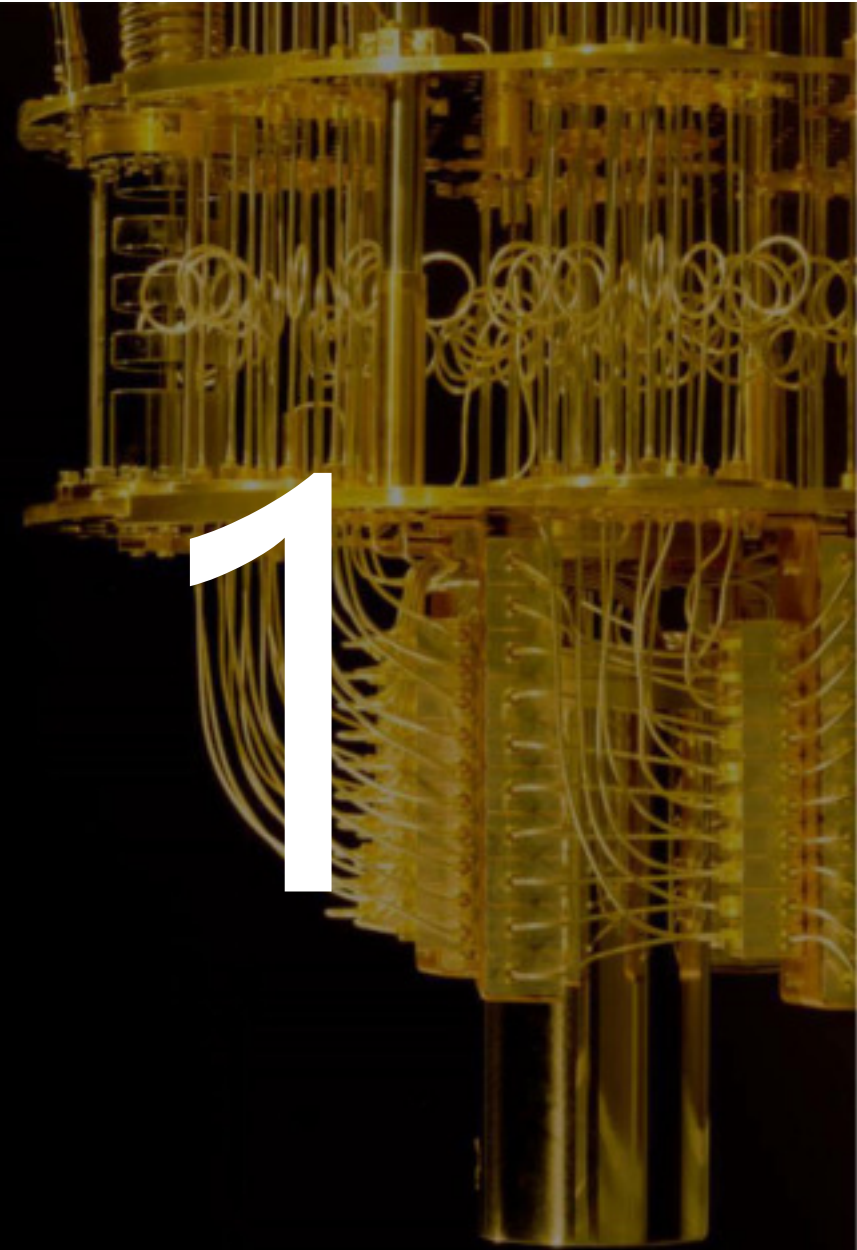
# Entanglement



Bits vs. Qubits

0

1

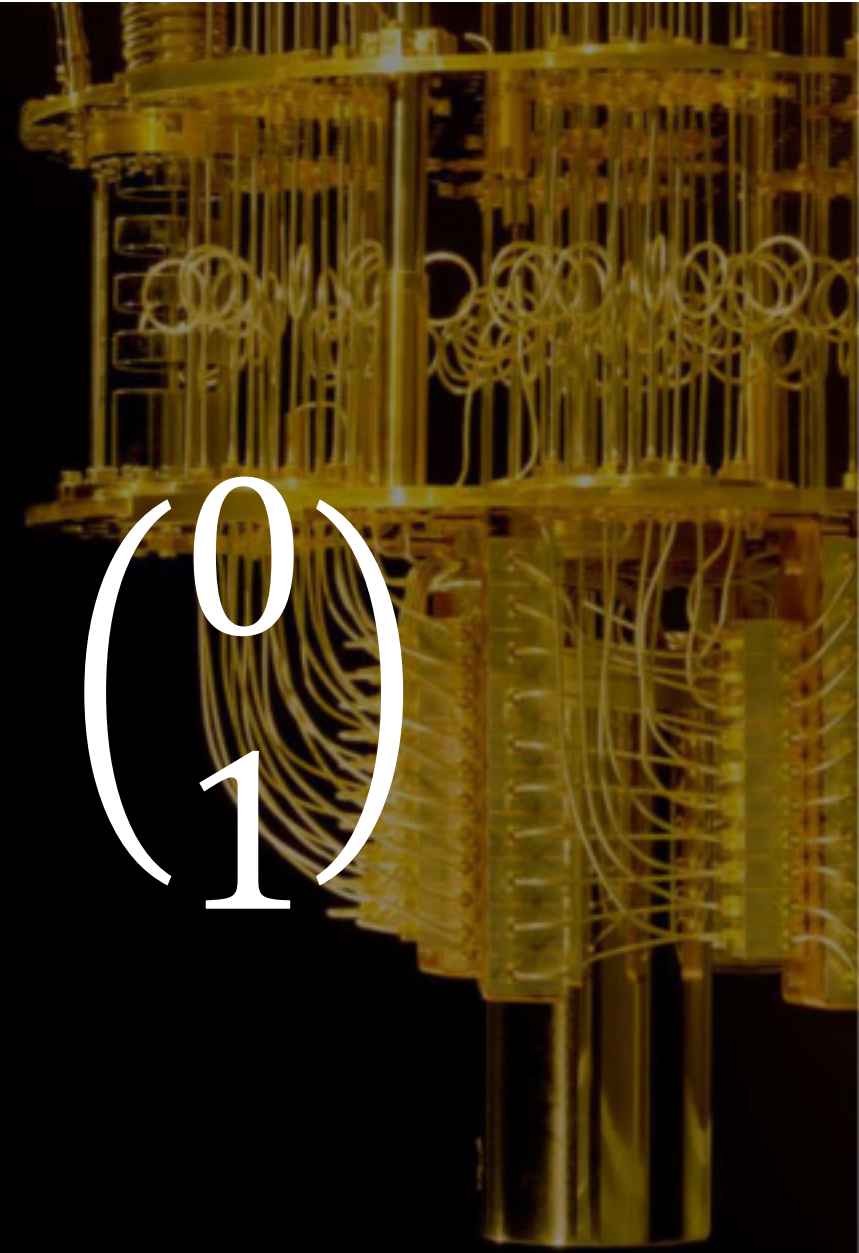




# Bits vs. Qubits

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$





# Bits vs. Qubits

Identity

$$f(x) = x$$

$$0 \rightarrow 0$$

$$1 \rightarrow 1$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Negation

$$f(x) = \neg x$$

$$0 \rightarrow 1$$

$$1 \rightarrow 0$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Constant-0

$$f(x) = 0$$

$$0 \rightarrow 0$$

$$1 \rightarrow 0$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Constant-1

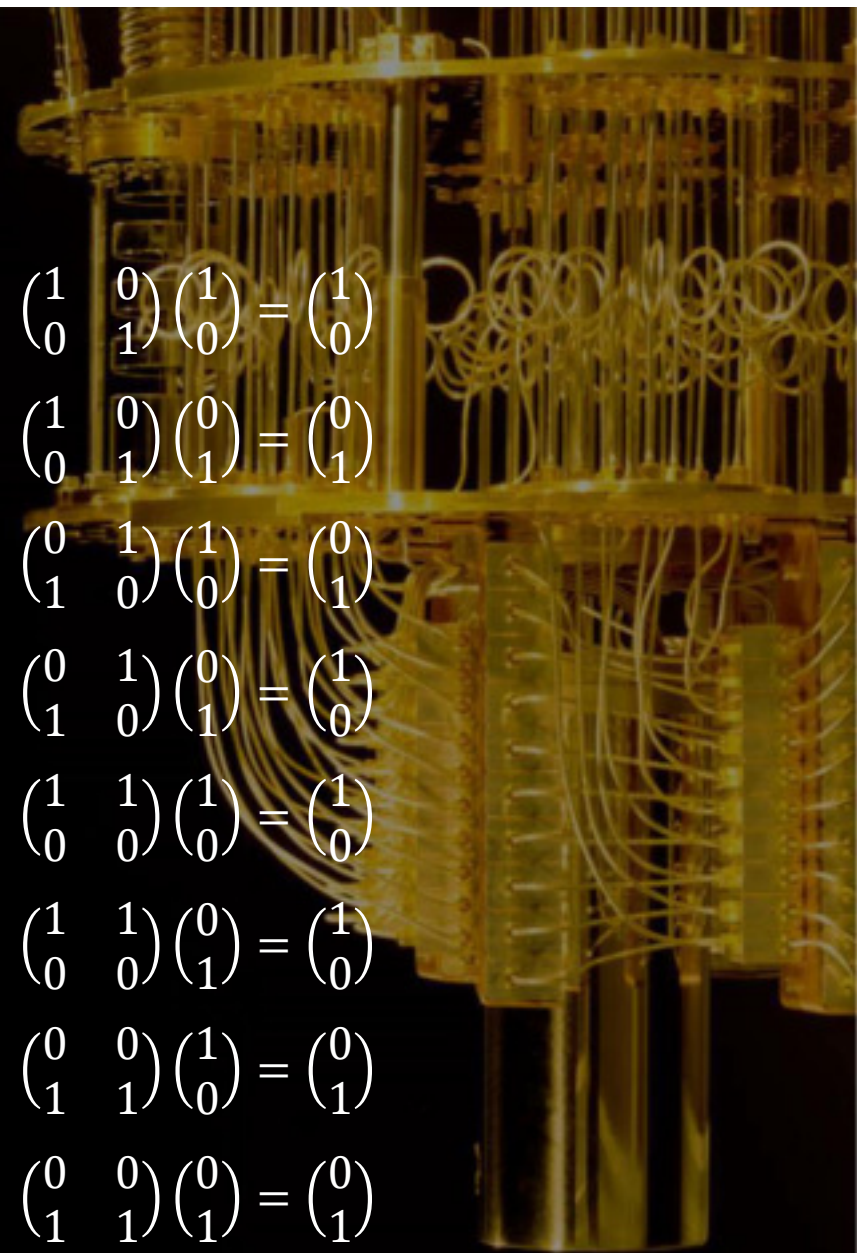
$$f(x) = 1$$

$$0 \rightarrow 1$$

$$1 \rightarrow 1$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

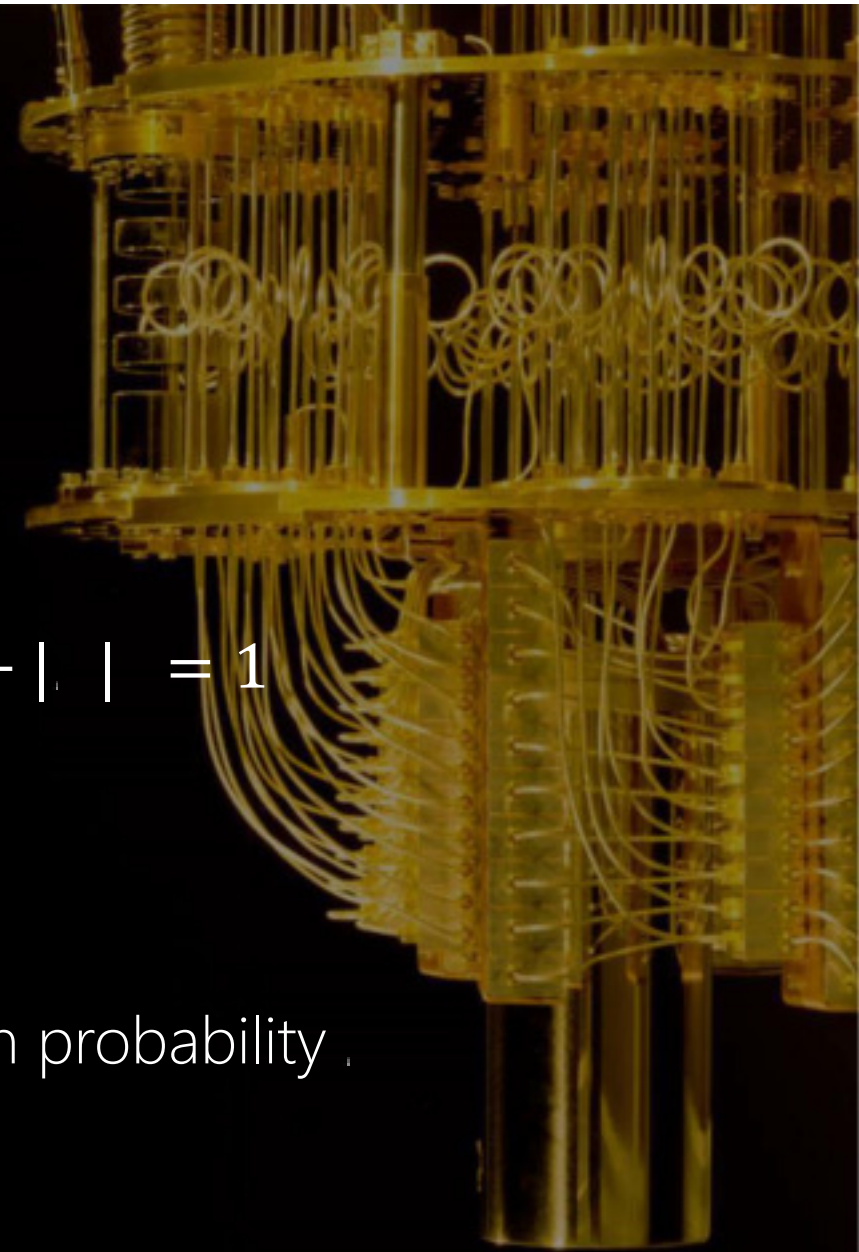


# Bits vs. Qubits

- Classical bit **0**, Quantum bit  $|0\rangle$
- Classical bit **1**, Quantum bit  $|1\rangle$
- Quantum bit in superposition

$$|0\rangle + \dots |1\rangle \text{ where } |a|^2 + |b|^2 = 1$$

- $a$  and  $b$  are complex number ( $a + bi$ )
- Value known after measurement
- Collapses to  $|0\rangle$  with probability  $|a|^2$  or  $|1\rangle$  with probability  $|b|^2$



# Bits vs. Qubits

- 2 Qubit system (4 values):

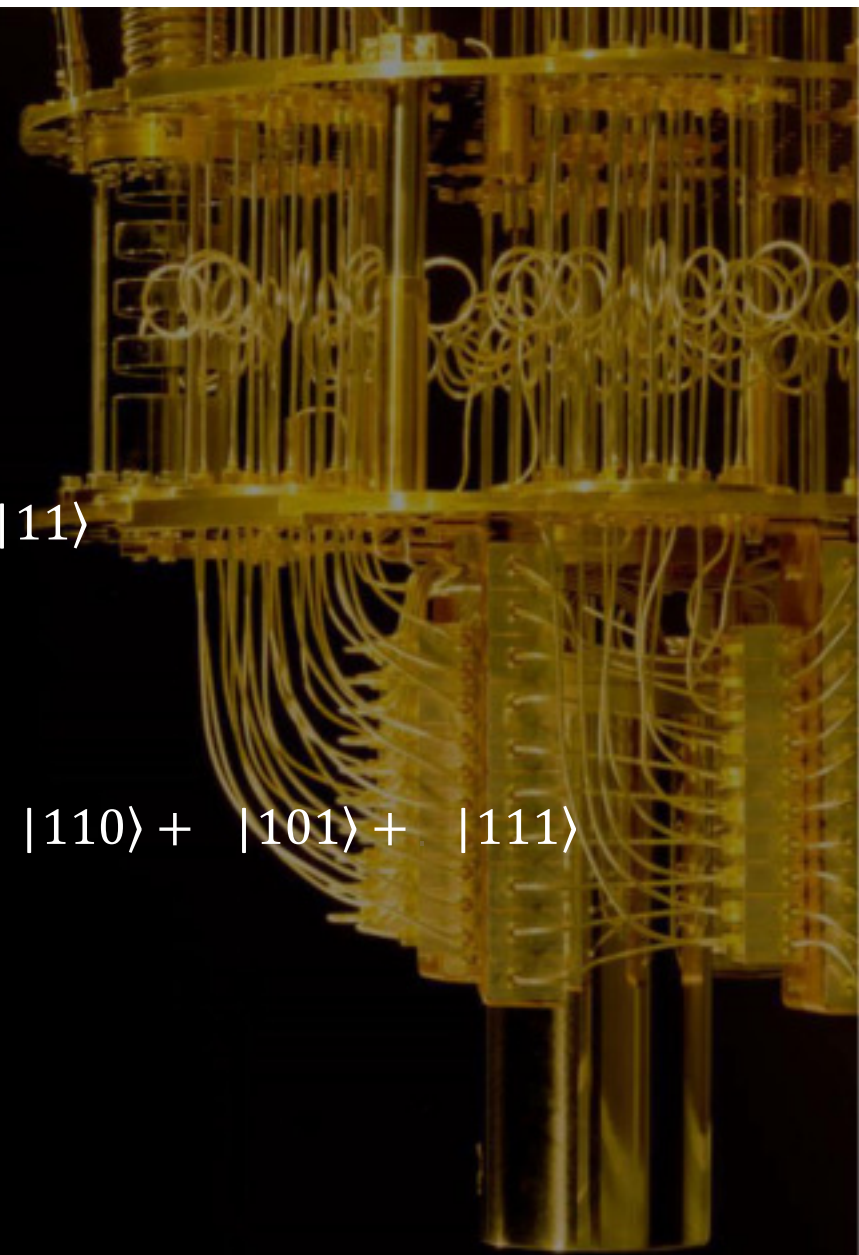
$$|00\rangle + |01\rangle + |10\rangle + |11\rangle$$

- 3 Qubit system (8 values):

$$|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |110\rangle + |101\rangle + |111\rangle$$

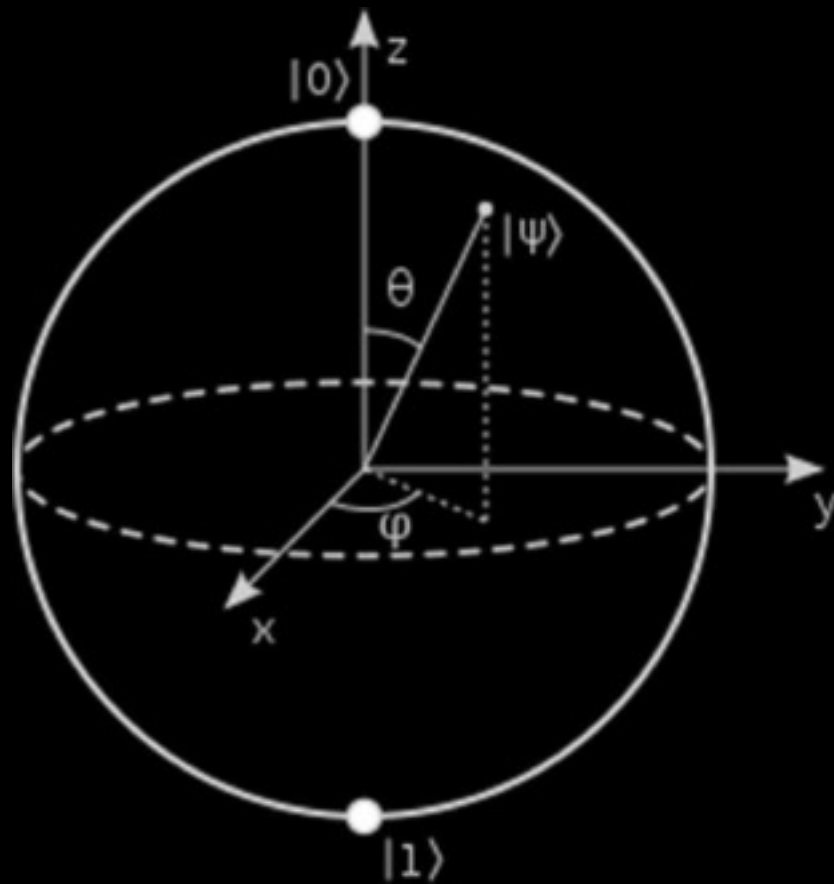
- 4 Qubit system (16 values):

...



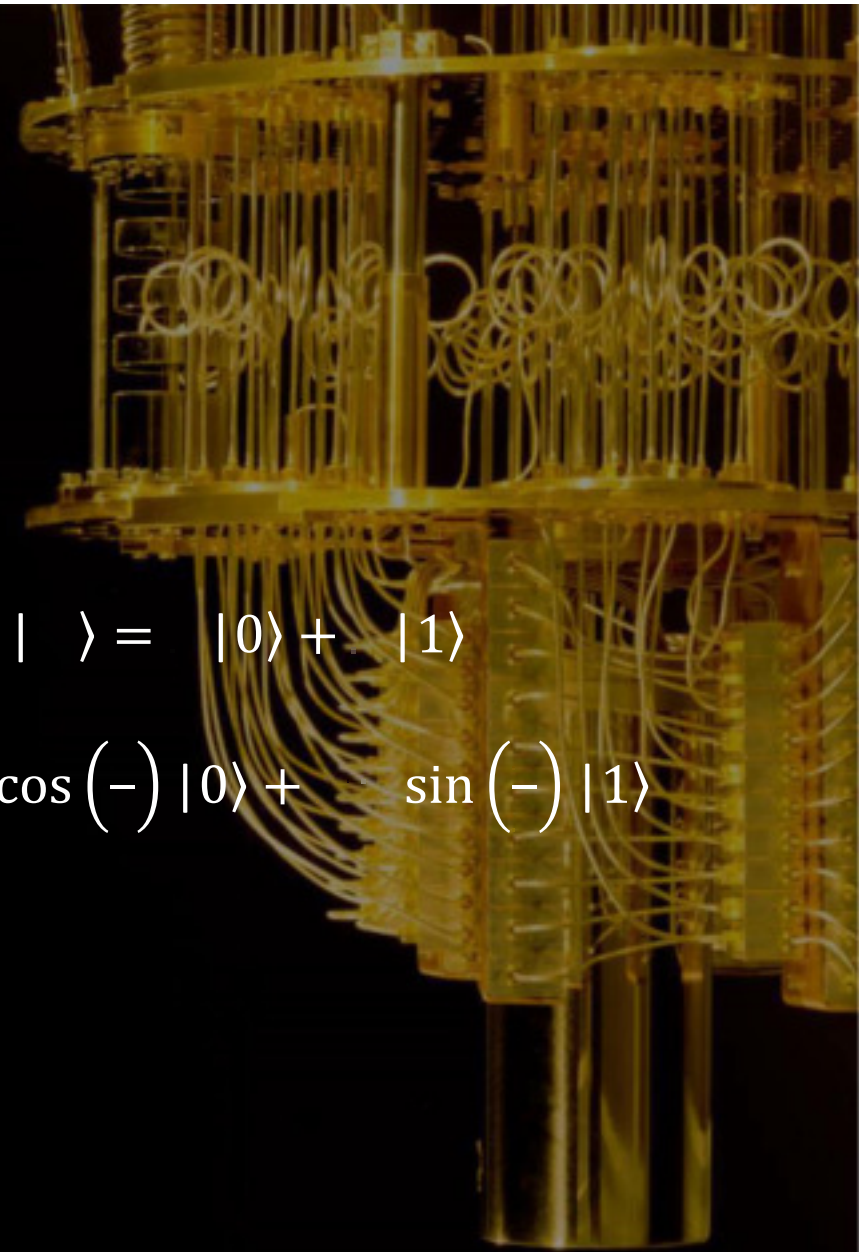


# Bits vs. Qubits

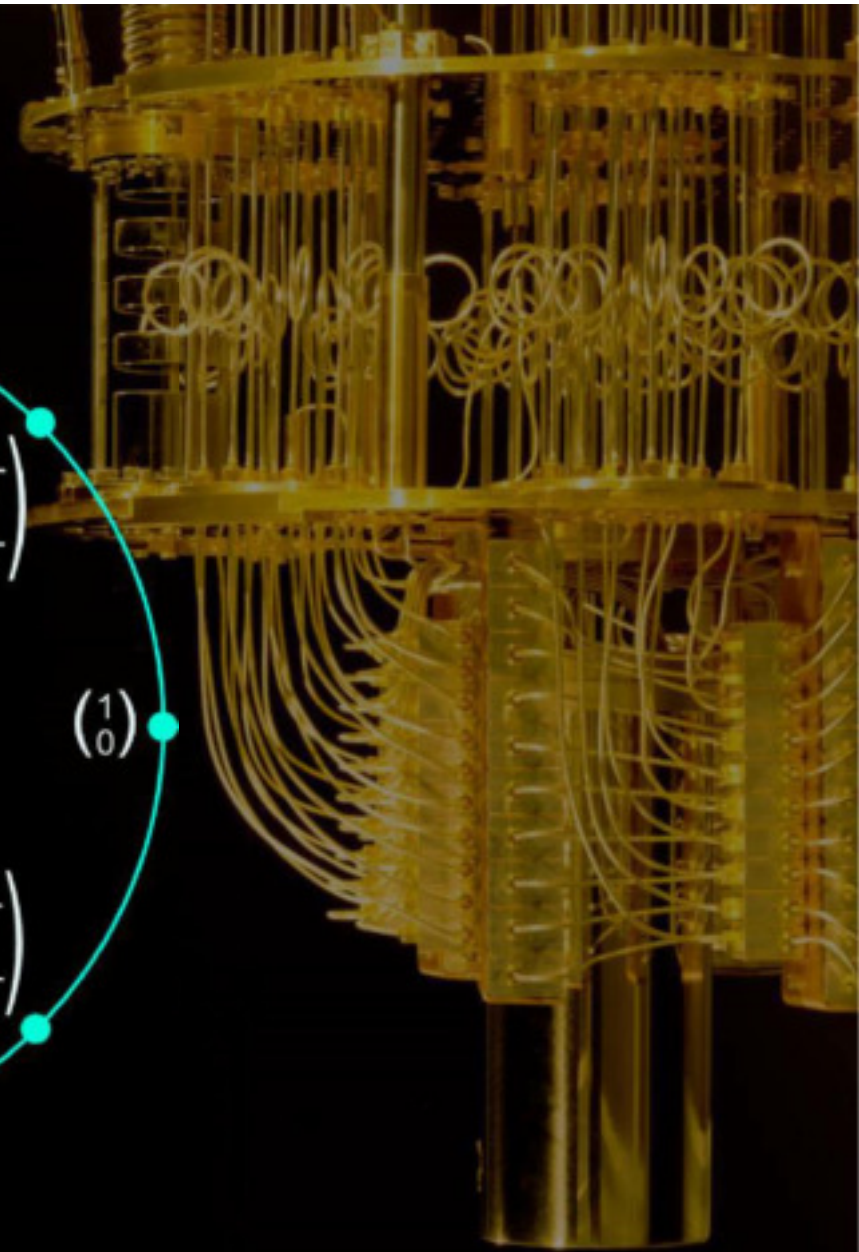
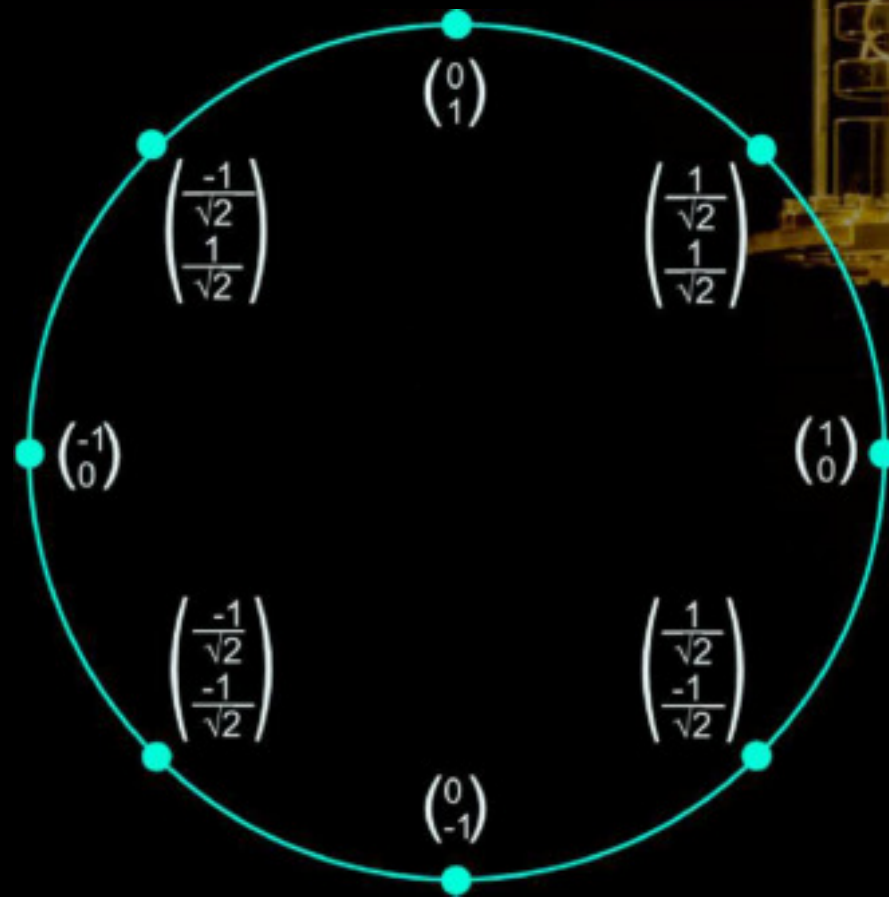


$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

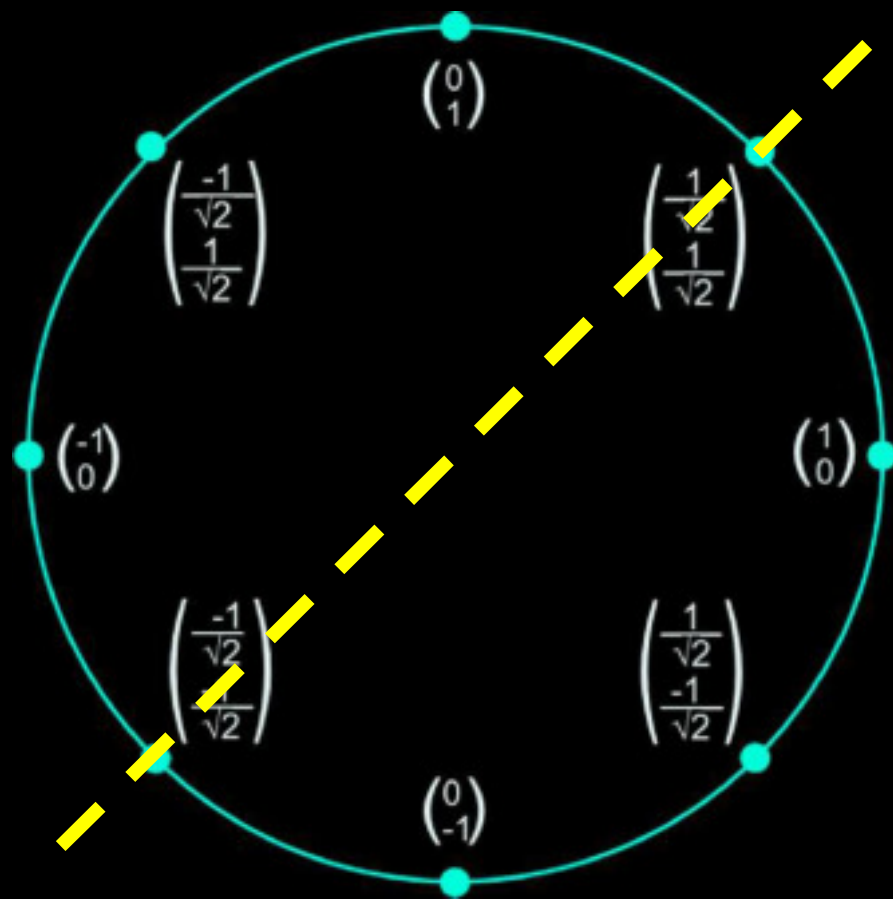
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$



# Bits vs. Qubits



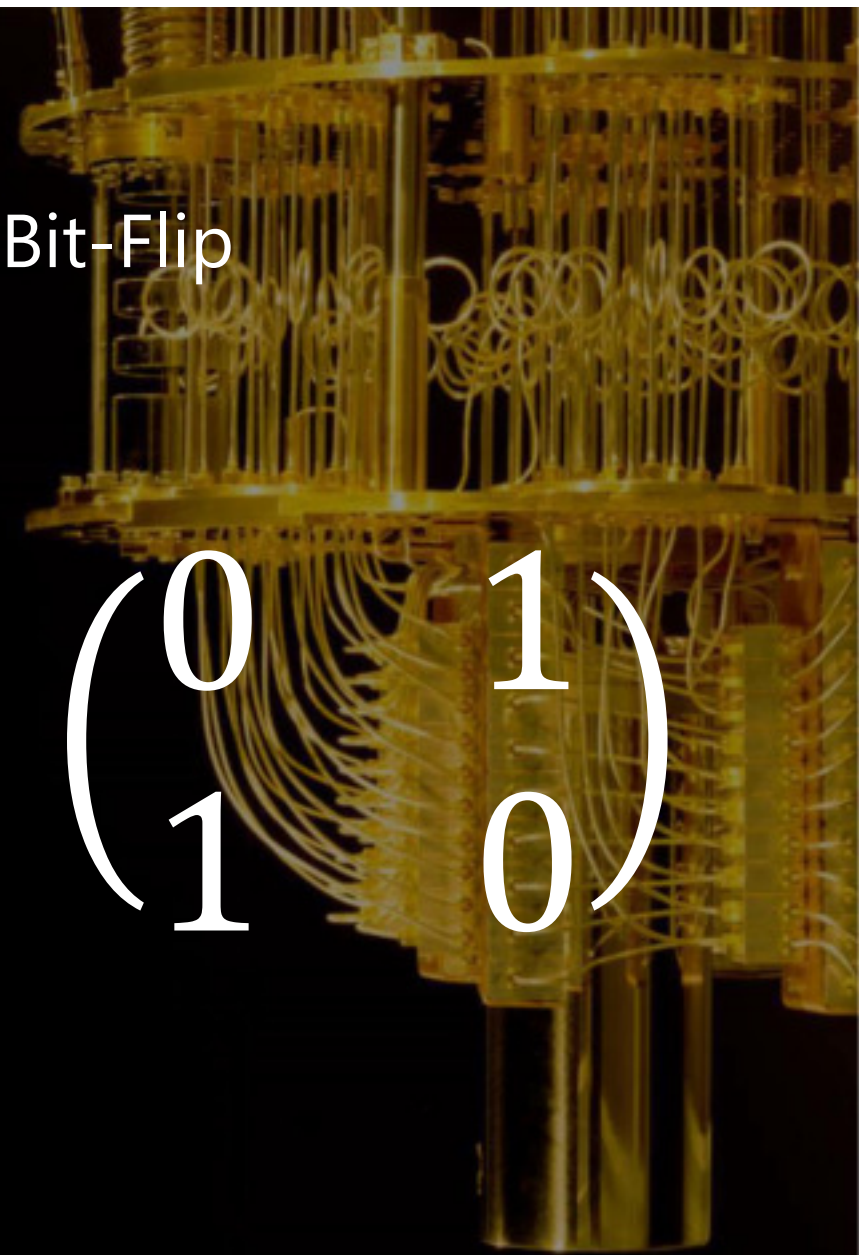
# Bits vs. Qubits



x

Bit-Flip

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

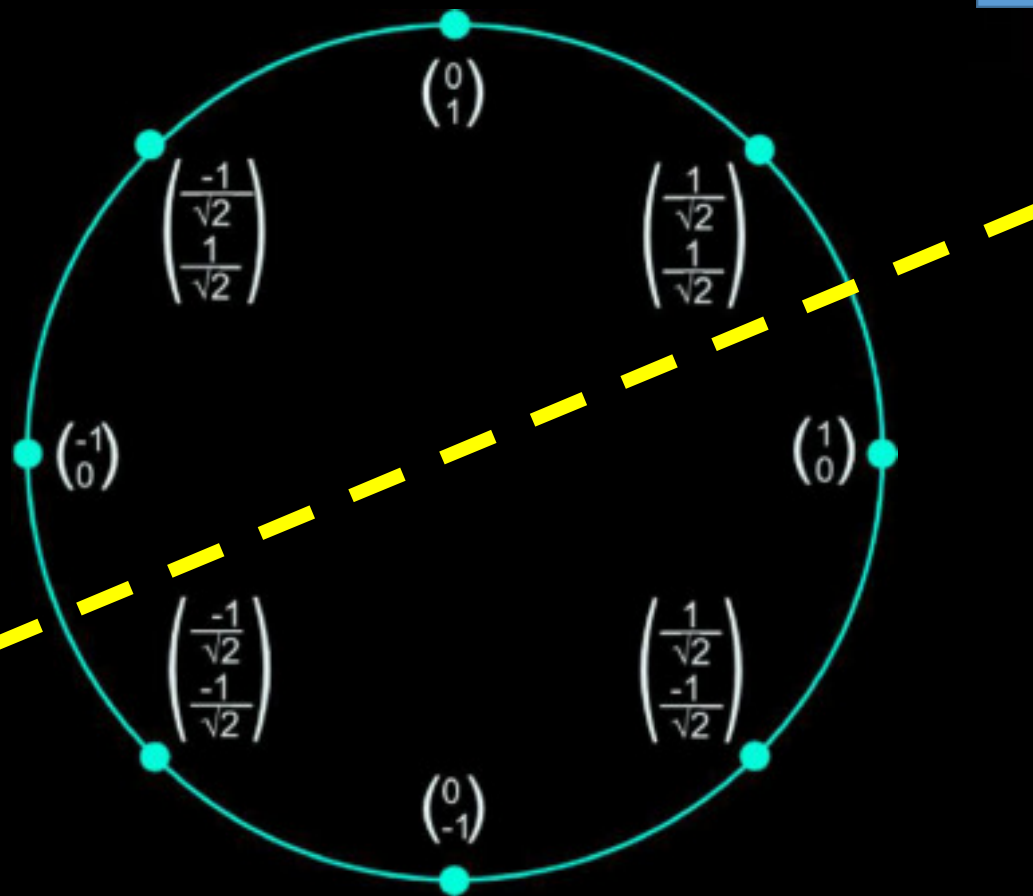




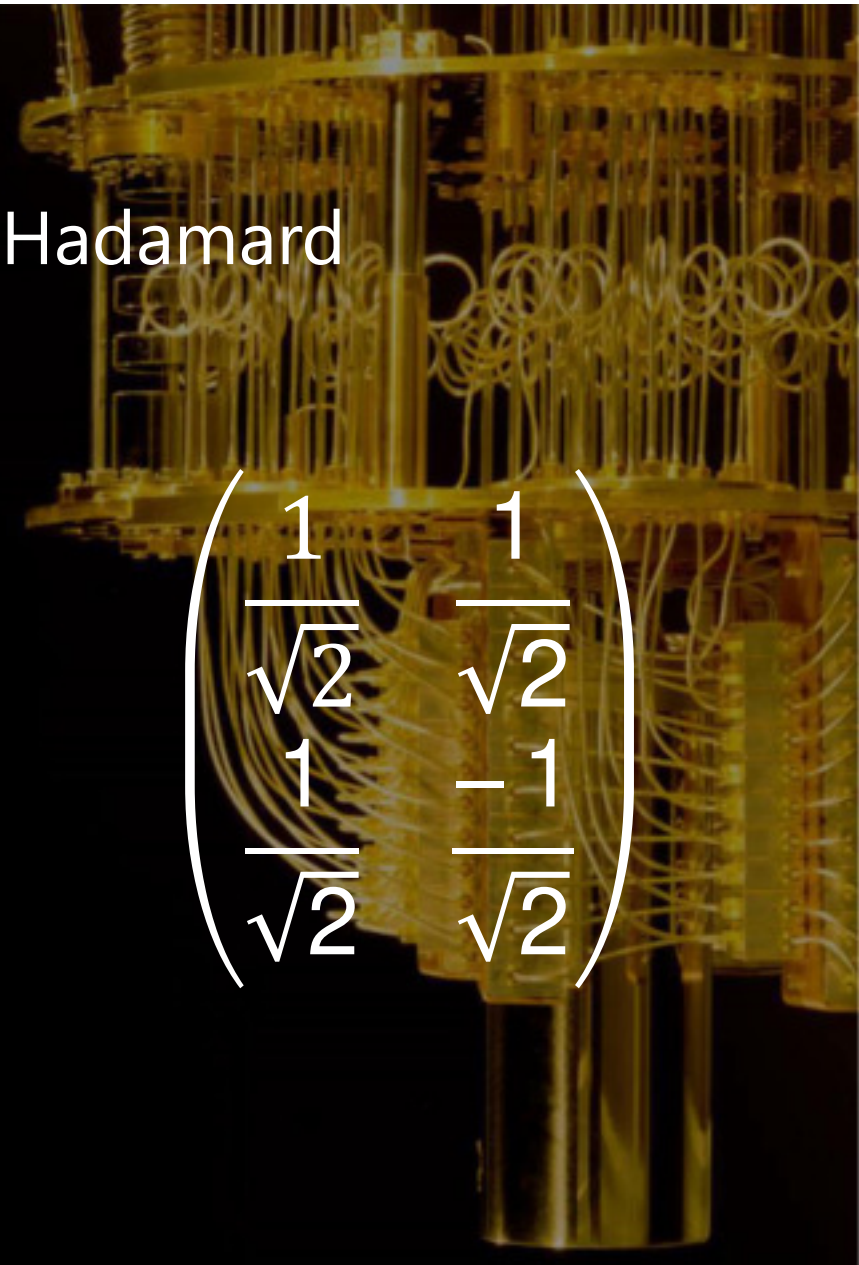
# Bits vs. Qubits

$H$

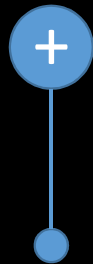
Hadamard



$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

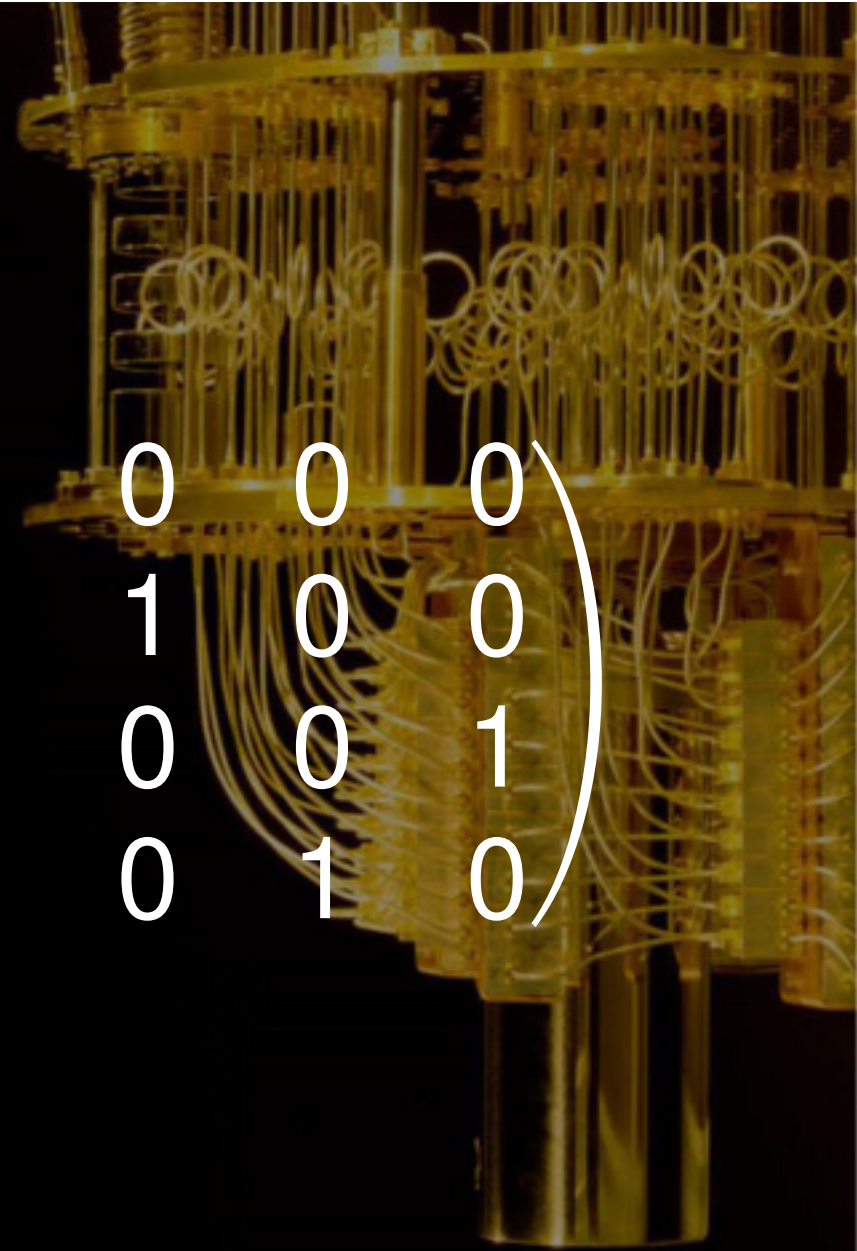


# Bits vs. Qubits



**CNOT**  
(2-qubit gate)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



# Bits vs. Qubits

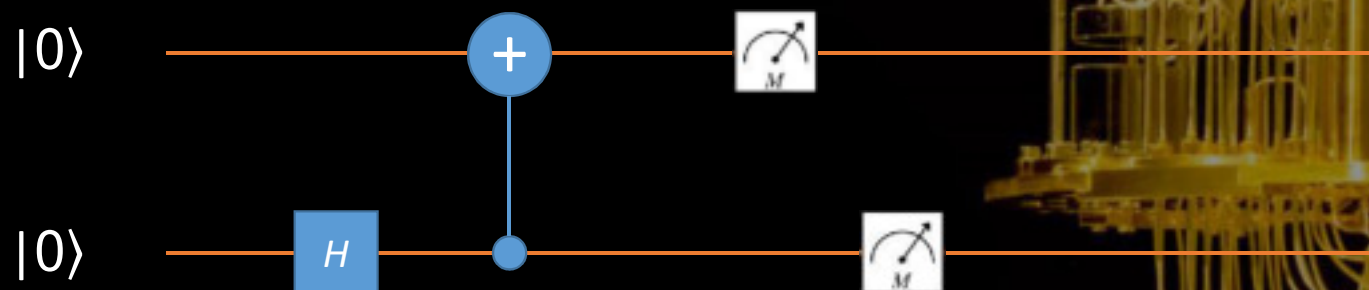


## Measurement

- Collapses a qubit to either  $|0\rangle$  or  $|1\rangle$
- A qubit in superposition has a 50% chance to collapse to  $|0\rangle$  and a 50% chance to collapse to  $|1\rangle$
- A measurement destroys any complex quantum state



# Entanglement



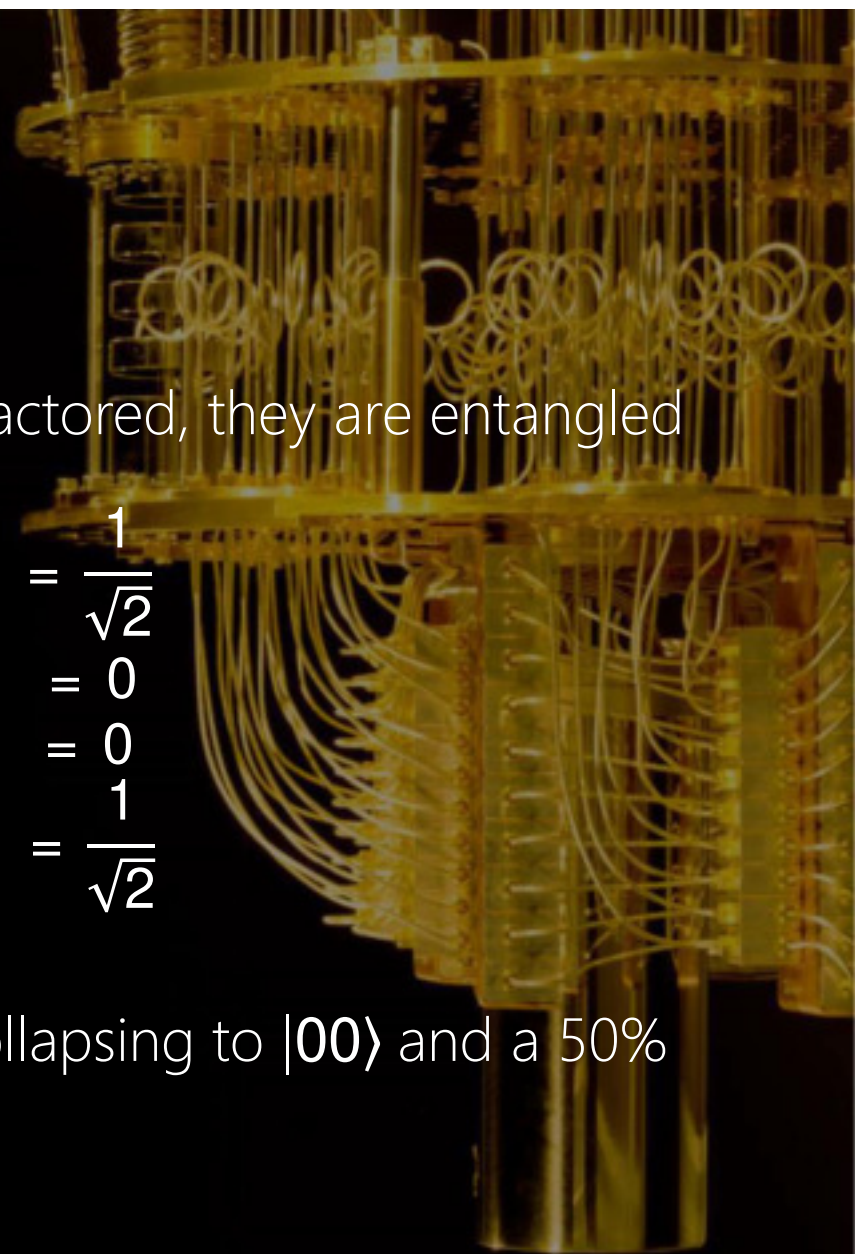
$$\begin{aligned}
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 1 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 1 \end{pmatrix} = ?
 \end{aligned}$$

# Entanglement

- If the product state of two qubits cannot be factored, they are entangled

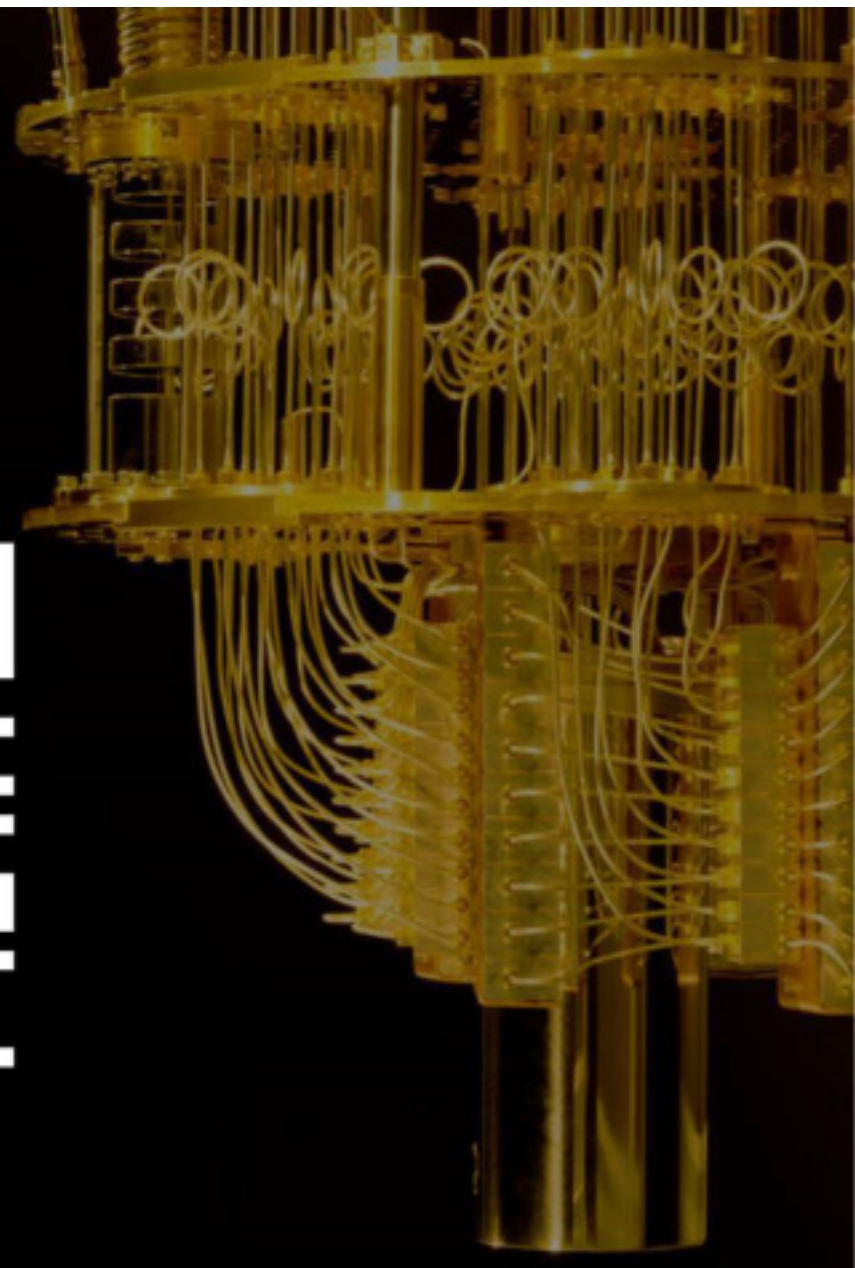
$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \end{pmatrix} \otimes \begin{pmatrix} \end{pmatrix} \rightarrow \begin{matrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{matrix}$$

- This set of two qubits has a 50% chance of collapsing to  $|00\rangle$  and a 50% chance of collapsing to  $|11\rangle$



# IBM Q Experience

- <https://quantumexperience.ng.bluemix.net>





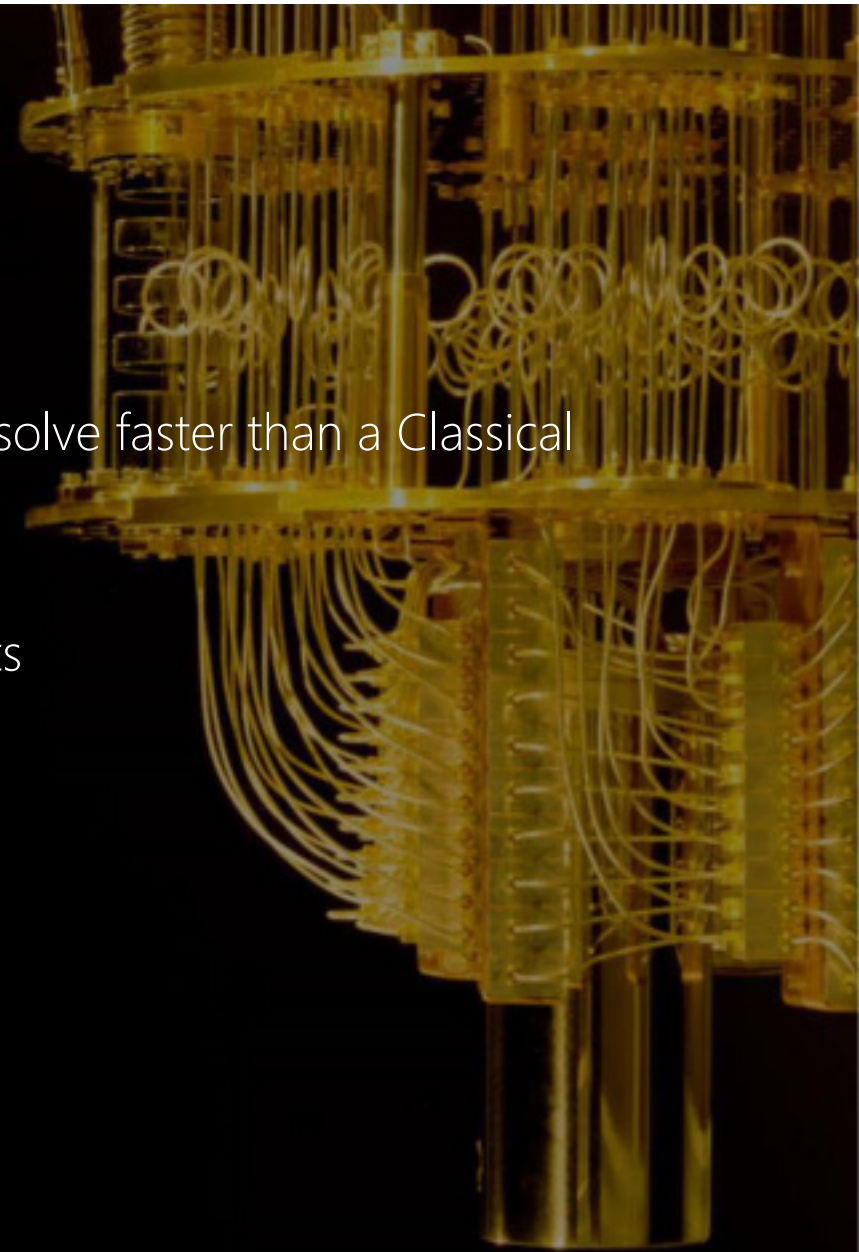
# Microsoft Q#

- <https://www.microsoft.com/en-us/quantum/development-kit>



# Quantum Algorithms

- Deutch (1985)
  - Is there a problem that a Quantum Computer can solve faster than a Classical Computer?
  - Deterministic!
- Deutsch–Jozsa (1992)
  - Based on Deutch (for 1 bit), but applicable for n-bits
  - Deterministic!
- Grover's algorithm (1996)
  - "Searching a database"
  - Probabilistic!
- Shor's algorithm (1994)
  - Prime factorization of large integers
  - Combination of classical and quantum algorithm
  - Probabilistic!



# Deutsch's algorithm

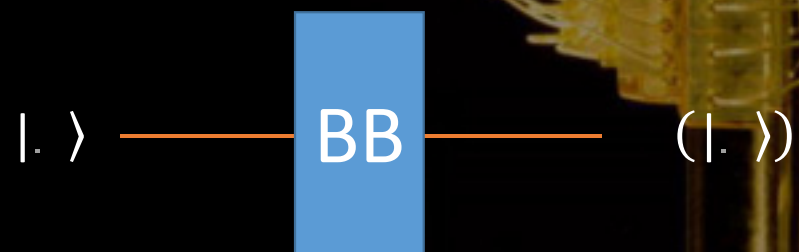
- Can a Quantum Computer be quicker than a Classical Computer?
- A Black-Box containing a function on one bit
- How many operations do you need to figure out the function if input and output is know?
  - On a Classical Computer?
  - On a Quantum Computer?





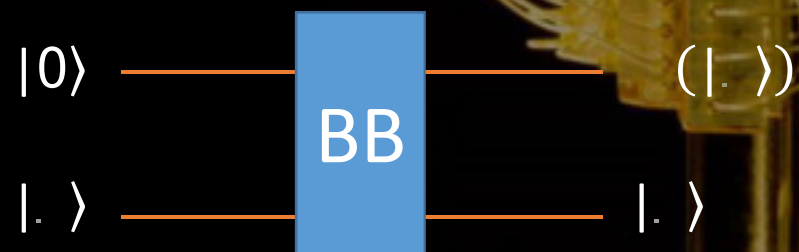
# Deutsch's algorithm

- It is important to ask the right question!
- A Black-Box containing a function on one bit
- How many operations do you need to figure out if the function is CONSTANT or VARIABLE if input and output is known?
  - On a Classical Computer?
  - On a Quantum Computer?

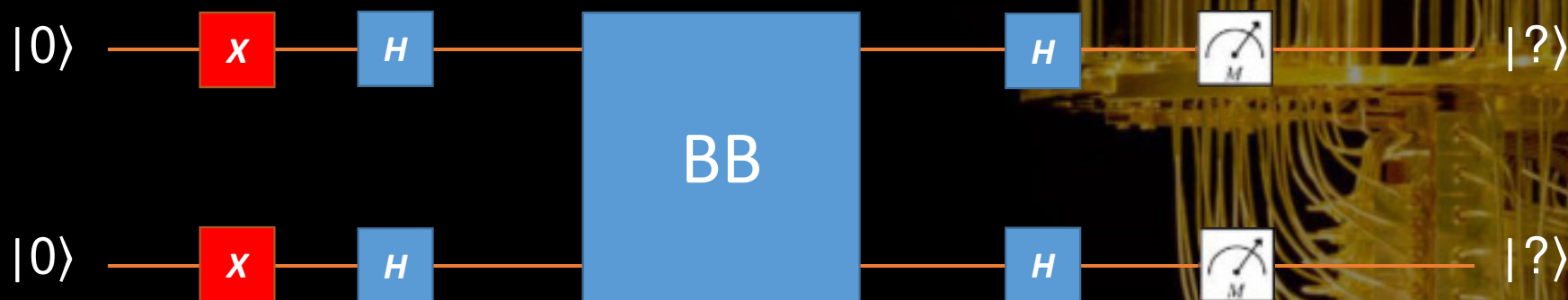


# Deutsch's algorithm

- It is important to ask the right question!
- A Black-Box containing a function on one bit
- How many operations do you need to figure out if the function is CONSTANT or VARIABLE if input and output is know?
  - On a Classical Computer?
  - On a Quantum Computer?



# Deutsch's algorithm

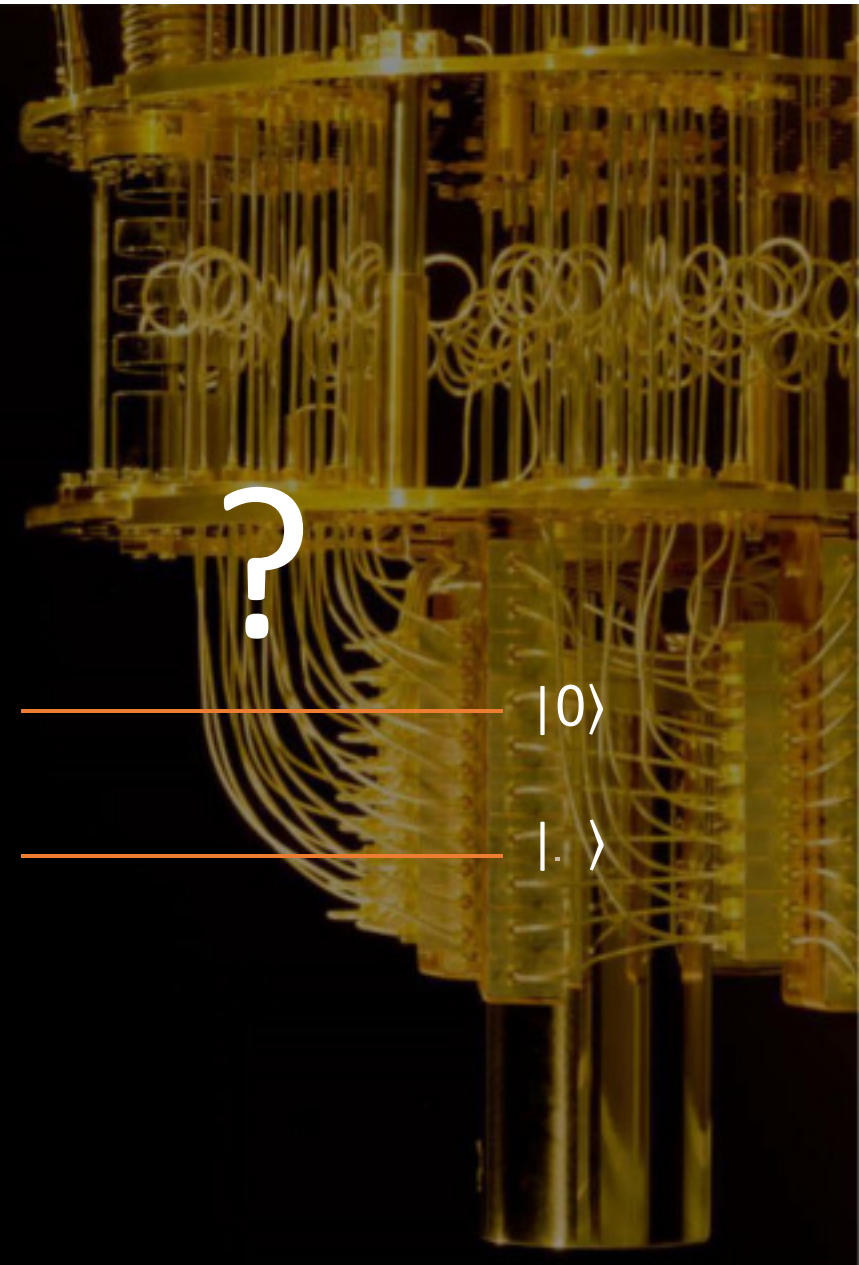
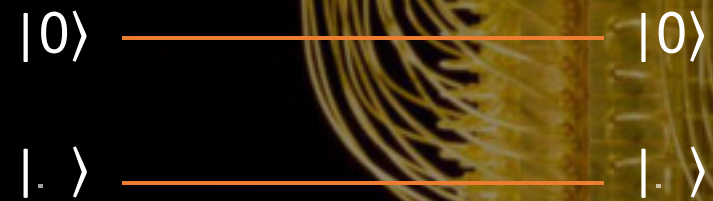
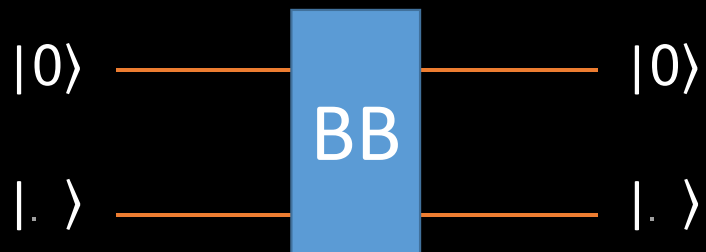


- If  $BB$  is a constant function  $\rightarrow$  Quantum state will always measure to  $|11\rangle$
- If  $BB$  is a variable function  $\rightarrow$  Quantum state will always measure to  $|01\rangle$



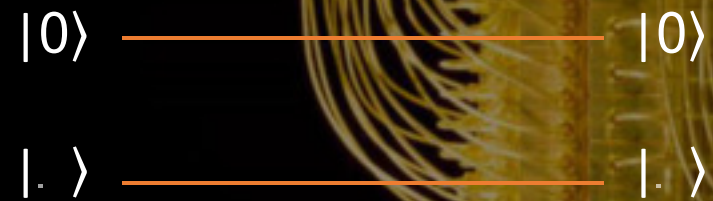
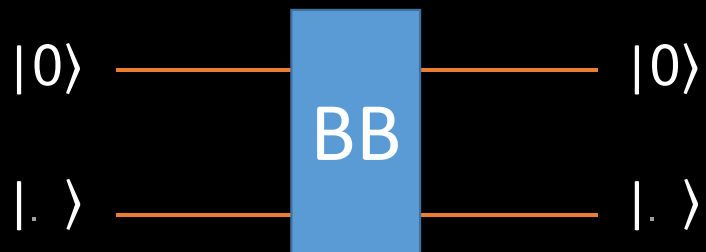
# Deutsch's algorithm

- Constant-0



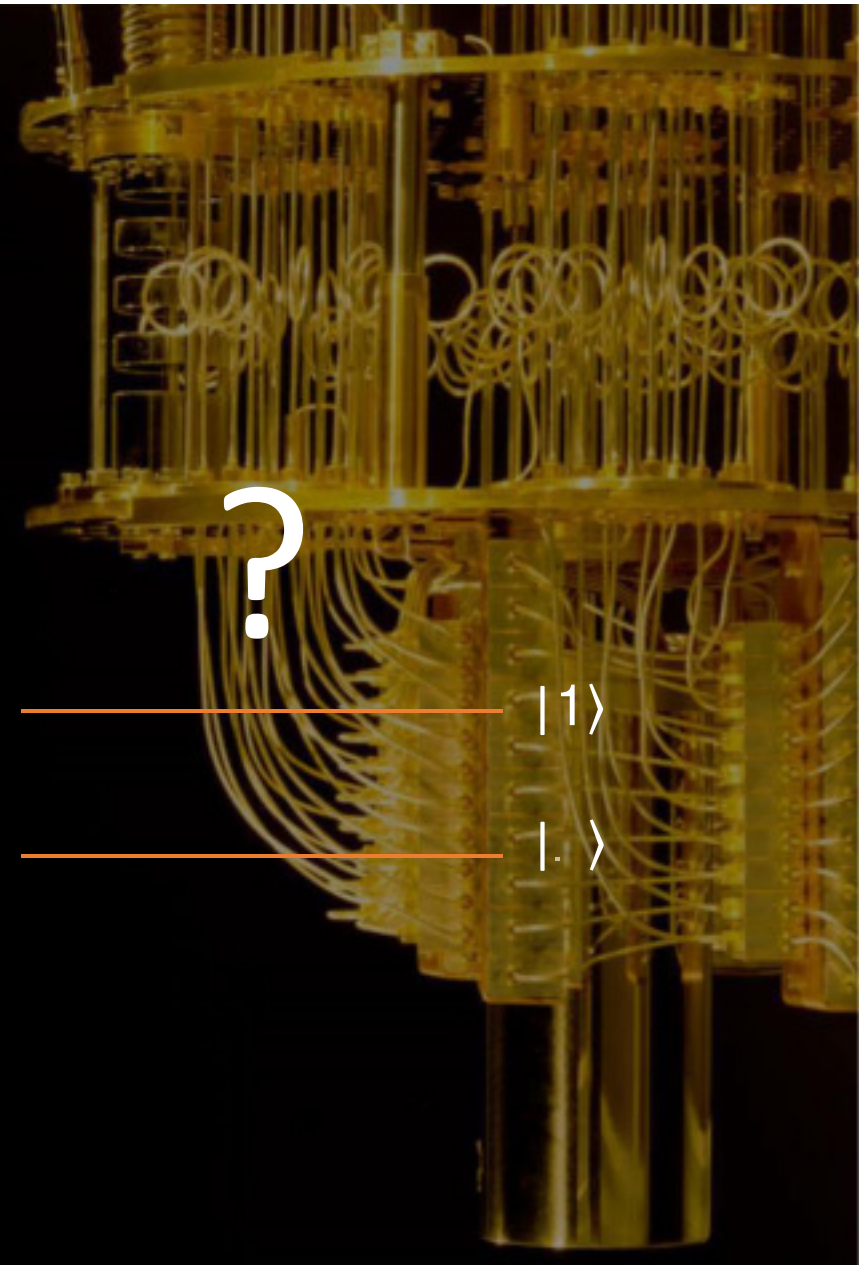
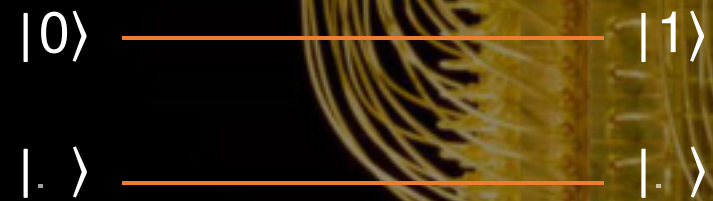
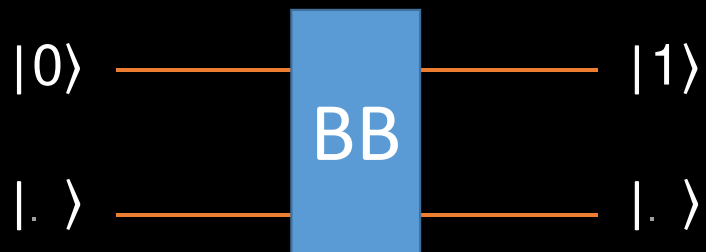
# Deutsch's algorithm

- Constant-0



# Deutsch's algorithm

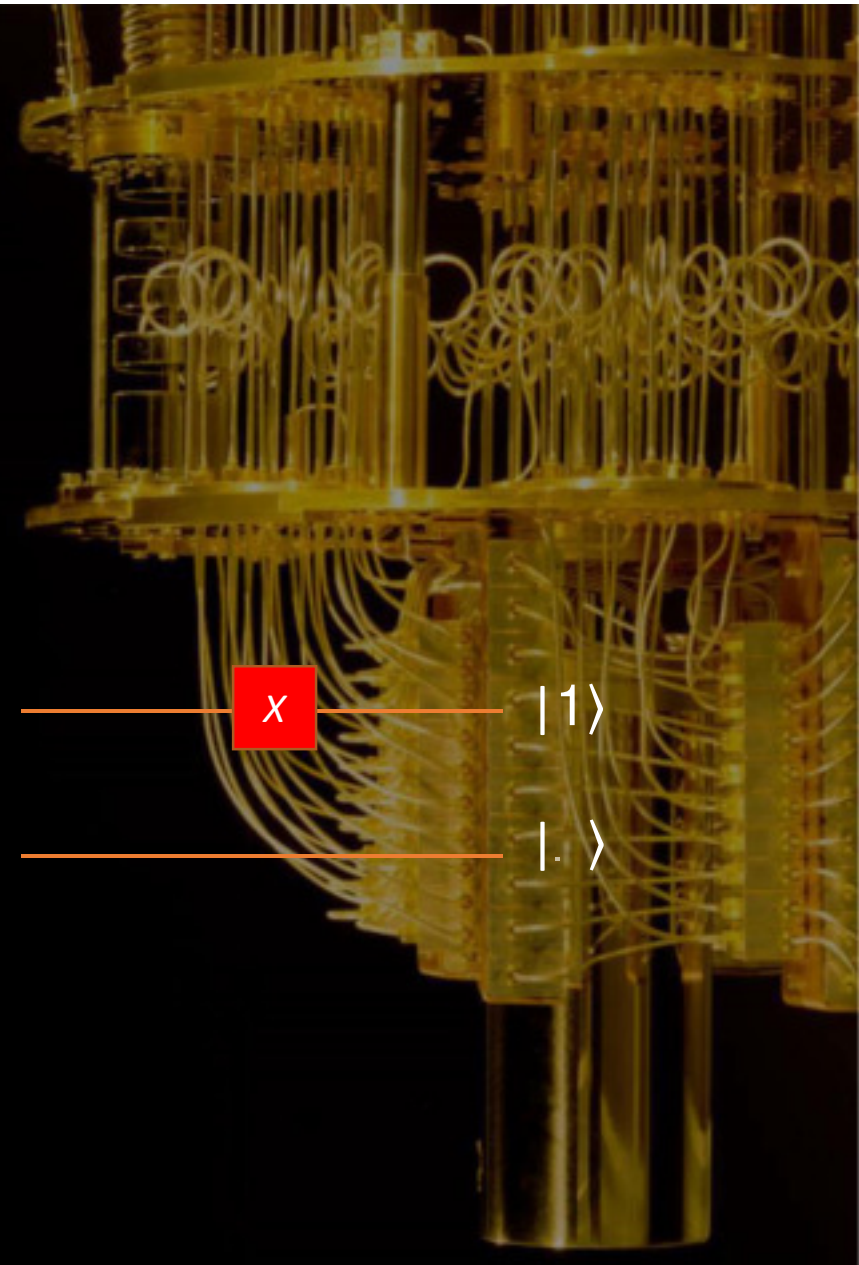
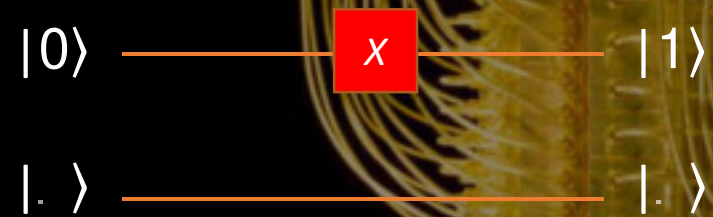
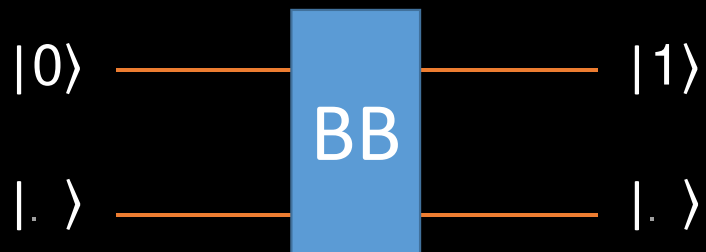
- Constant-1





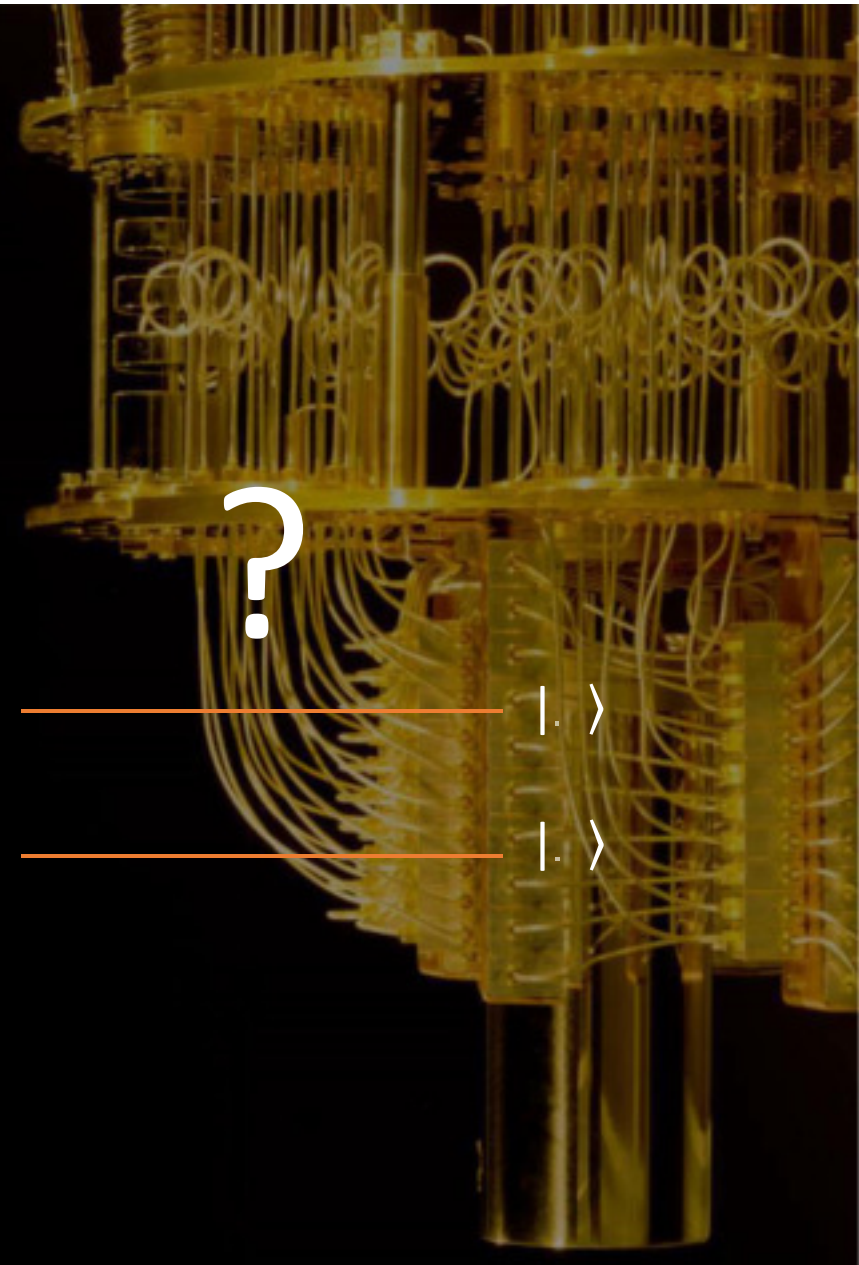
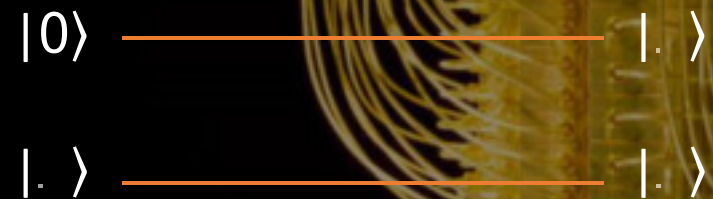
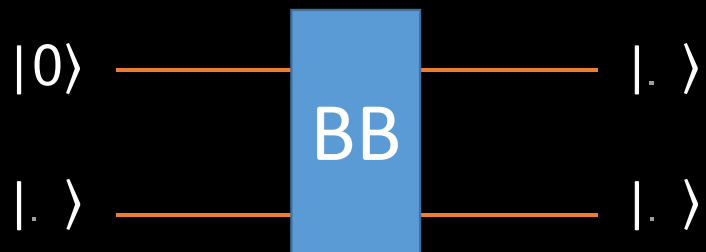
# Deutsch's algorithm

- Constant-1



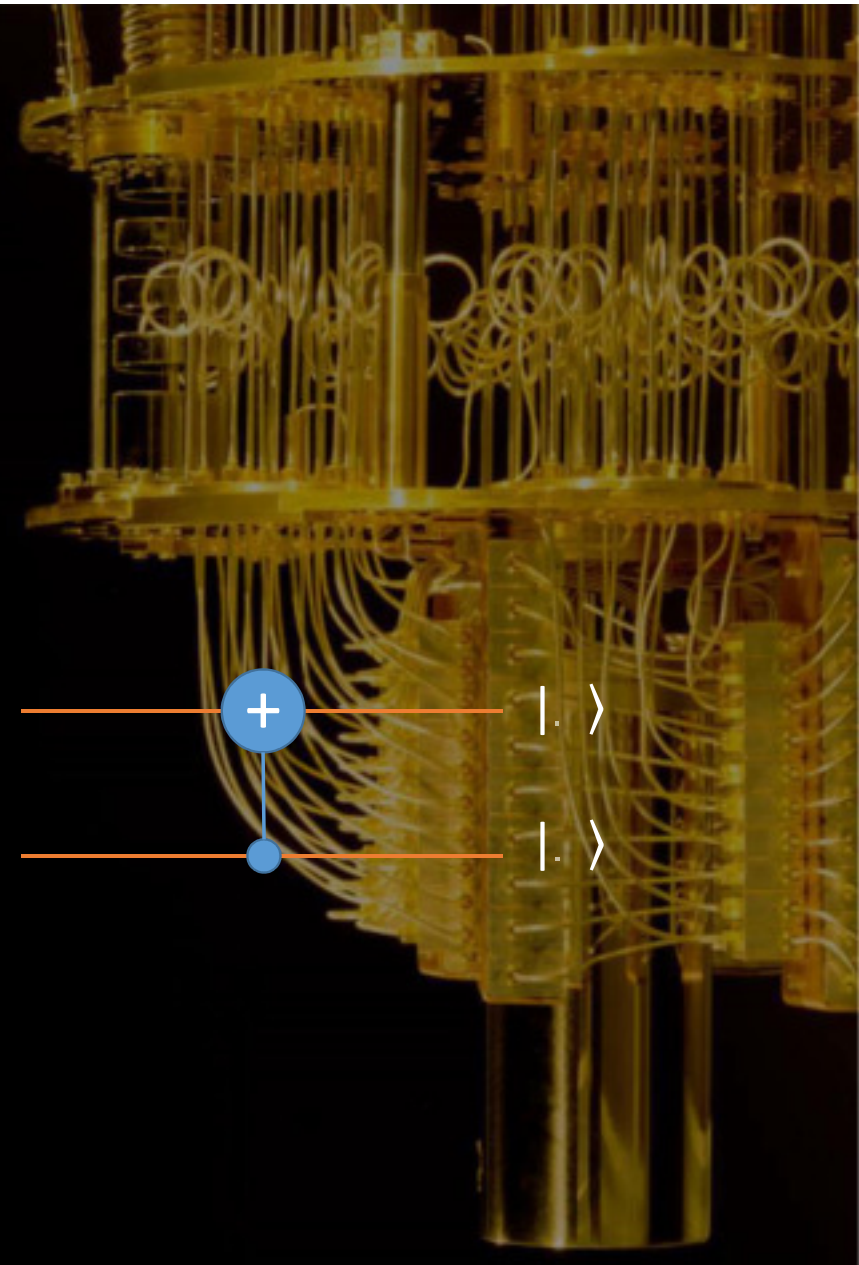
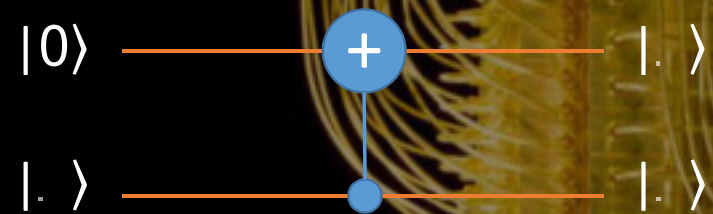
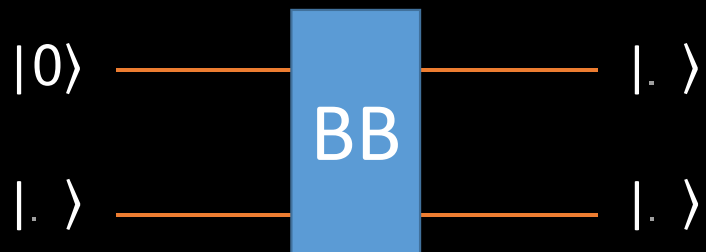
# Deutsch's algorithm

- Identity



# Deutsch's algorithm

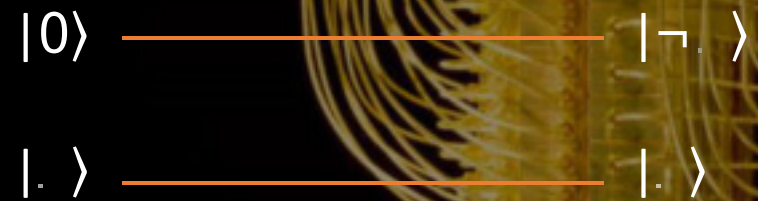
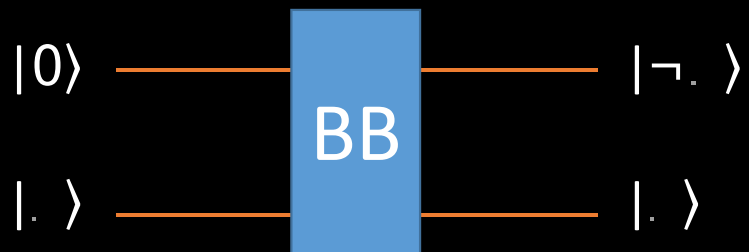
- Identity





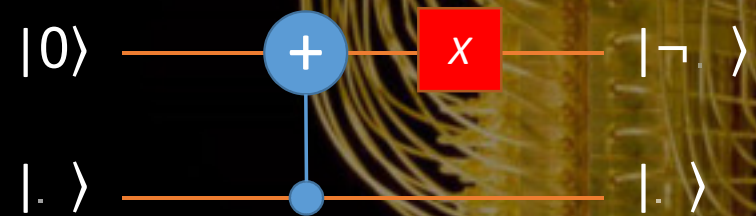
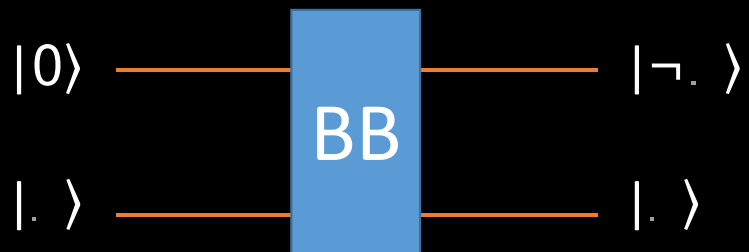
# Deutsch's algorithm

- Negation



# Deutsch's algorithm

- Negation



# Deutsch's algorithm

- Constant-0 (circuit overview)





# Deutsch's algorithm

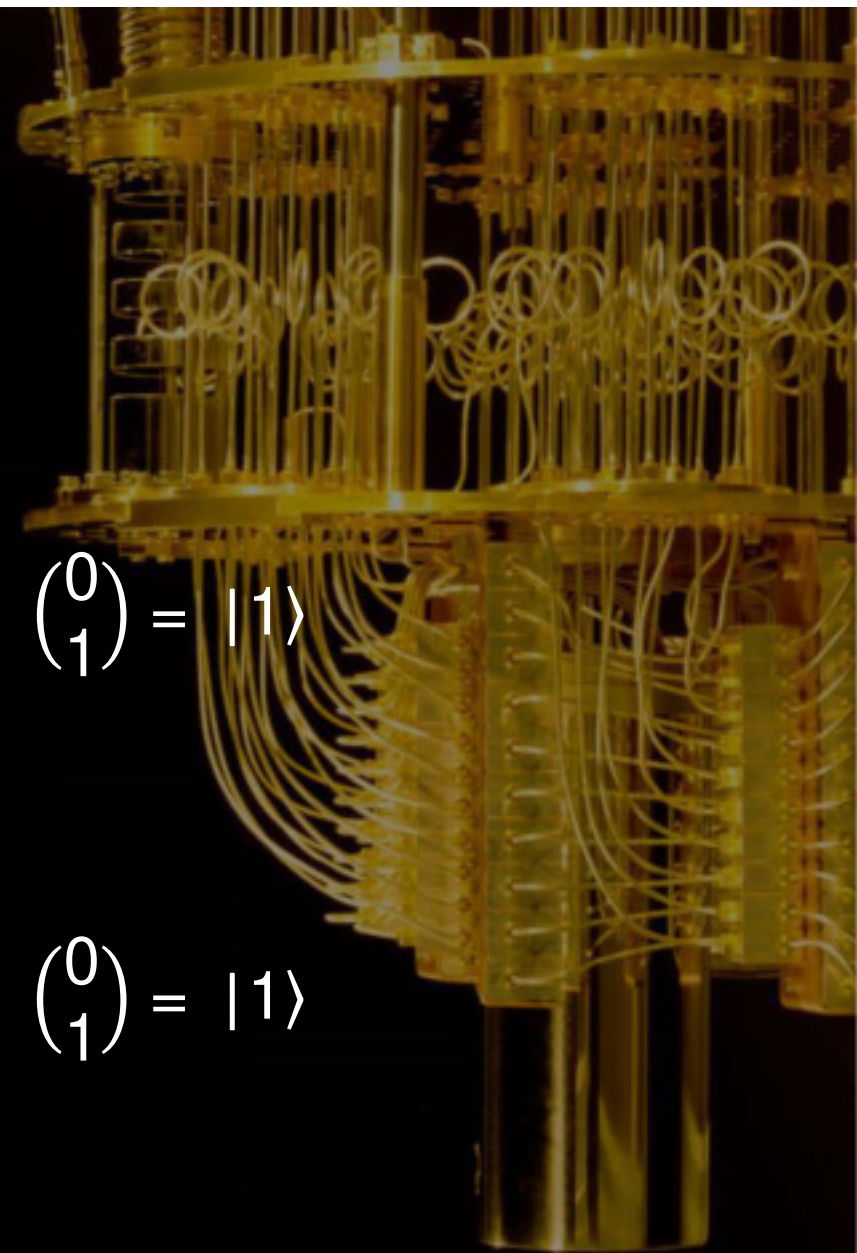
- Constant-0 (calculated proof – part 1)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \sqrt{2} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \sqrt{2} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

# Deutsch's algorithm

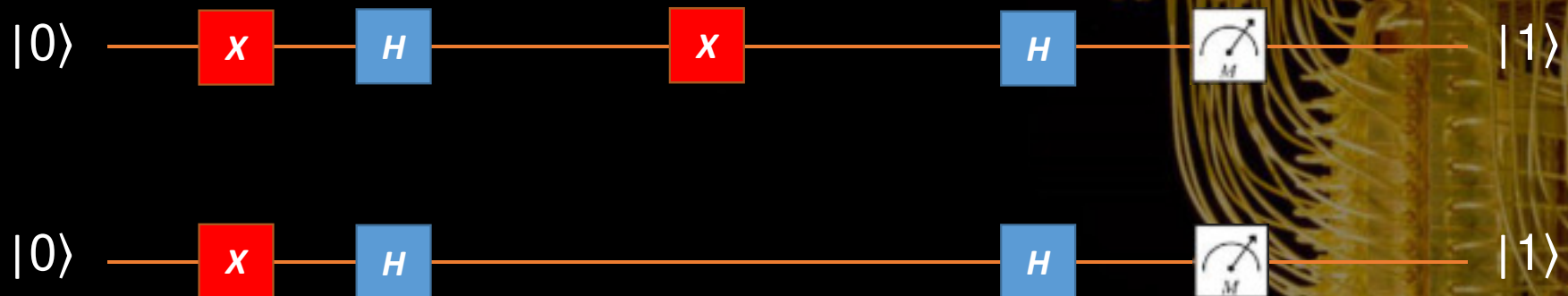
- Constant-0 (calculated proof – part 2)

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$
$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$



# Deutsch's algorithm

- Constant-1 (circuit overview)





# Deutsch's algorithm

- Constant-1 (calculated proof – part 1)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \sqrt{2} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \sqrt{2} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

# Deutsch's algorithm

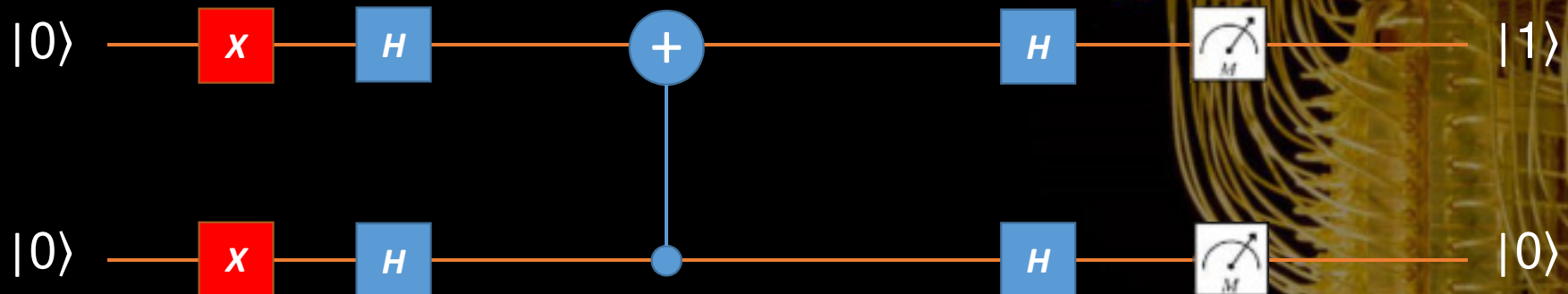
- Constant-1 (calculated proof – part 2)

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

# Deutsch's algorithm

- Identity (circuit overview)





# Deutsch's algorithm

- Identity (calculated proof – part 1)

$$\begin{aligned} |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \\ |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{-1}{2} \end{pmatrix} \end{aligned}$$

# Deutsch's algorithm

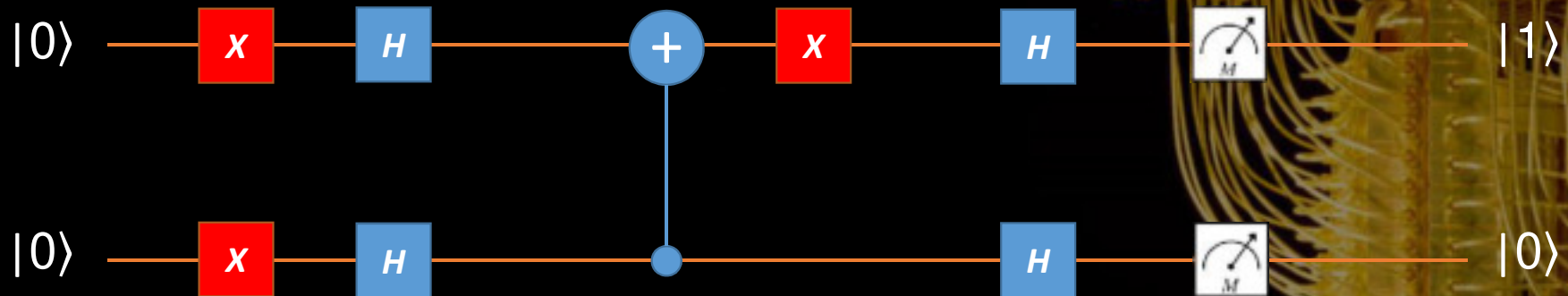
- Identity (calculated proof – part 2)

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

# Deutsch's algorithm

- Negation (circuit overview)





# Deutsch's algorithm

- Negation (calculated proof – part 1)

$$\begin{aligned} |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \\ |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{-1}{2} \end{pmatrix} \end{aligned}$$

# Deutsch's algorithm

- Negation (calculated proof – part 2)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$



*“About your cat, Mr. Schrödinger – I have good news and bad news.”*



The background of the slide features a complex network of blue lines and dots, resembling a molecular structure or a data network, set against a dark blue gradient.
$$\begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \text{Johnny Hooyberghs} \\ \text{Robin Vercammen} \end{pmatrix}$$

johnny.hooyberghs@involved-it.be  
@djohnnieke

robin.vercammen@involved-it.be  
@Robin\_Vercammen

<https://github.com/Djohnnie/QuantumComputingQSharpIntroduction2018>

[www.involved-it.be](http://www.involved-it.be)